工程數學 (四) - 期末考 (閉書)

18:00-22:00, 25/5, 1995

I. The auxilliary system $U(x,s;t,\tau)$, which is a solution of

$$\mathcal{L}\{U\} = \frac{\partial^2 U(x,s;t,\tau)}{\partial t^2} - c^2 \frac{\partial^2 U(x,s;t,\tau)}{\partial x^2} = \delta(x-s)\delta(t-\tau), -\infty < x < \infty, t > 0$$

with initial conditions

$$\lim_{t\to\tau} U(x,s;t,\tau) = 0$$

$$\lim_{t\to\tau} \dot{U}(x,s;t,\tau) = 0$$

and no boundary condition since $-\infty < x < \infty$,

I.(a) Given the exact form for $U(x,s;t,\tau)$ (30 %) :

$$U(x,s;t,\tau) = \frac{1}{2c} H(x-s+c(t-\tau)) - \frac{1}{2c} H(x-s-c(t-\tau)) = \bar{U}(x-s,t-\tau)$$

Answer the following questions and explain the reason.

$$U(x,s;t,\tau) = \ \mathit{or}\ \neq U(s,x;t,\tau) \tag{1}$$

$$U(x,s;t,\tau) = \ \mathit{or}\ \neq U(x,s;\tau,t) \tag{2}$$

$$U(x,s;t,\tau) = \ \mathit{or}\ \neq U(s,x;\tau,t) \tag{3}$$

$$\mathcal{L}\{U(x,s;t,\tau)\} = \ \mathit{or}\ \neq \mathcal{L}\{U(s,x;\tau,t)\} \tag{4}$$

$$U(s,x;\tau^+,\tau) = 0 \ \mathit{or}\ U(s,x;\tau,\tau^+) = 0 \tag{5}$$

$$\frac{\partial^2 U(s,x;t,\tau)}{\partial x^2} = \ \mathit{or}\ \neq \frac{\partial^2 U(s,x;t,\tau)}{\partial s^2} \tag{6}$$

$$\frac{\partial^2 U(s,x;t,\tau)}{\partial t^2} = \ \mathit{or}\ \neq \frac{\partial^2 U(s,x;t,\tau)}{\partial \tau^2} \tag{7}$$

I.(b) Series form (30 %):
Derivation of $p_m(x)$:

$$\delta(x-s) = \sum_{m=1}^{\infty} p_m(x) sin(m\pi s/l)$$

Derivation of $q_m(t,\tau)$:

$$\mathbf{U}(\mathbf{x},\mathbf{s};t,\tau) = \sum_{m=1}^{\infty} q_m(t,\tau)\ sin(m\pi x/l) sin(m\pi s/l)$$

Repeat Eqs.(1) to (7) and explain the reason.

II. Solve the heat conduction problem with time-dependent boundary conditions (30%) :

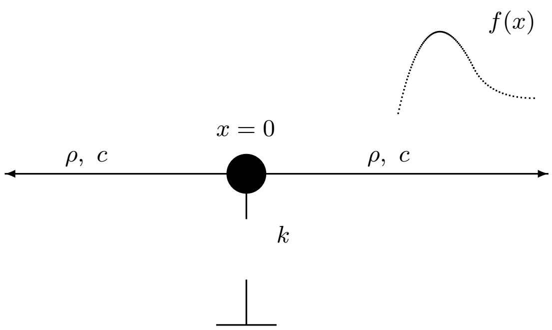
$$\frac{\partial u(x,t)}{\partial t} - \alpha^2 \frac{\partial^2 u(x,t)}{\partial x^2} = 0$$

with initial conditions

$$u(x,0) = 0$$

and boundary conditions

$$u(0,t) = a(t)$$
$$u(l,t) = b(t)$$



(20,12,19,11.5,20,11) (20,11,21,10.5,20,10) (20,10,19,9.5,20,9) (20,9,21,8.5,20,8)

Fig.1 An infinite string with a spring and a lump mass at $x = 0$

III. As shown in Fig.1, string reflection and transmission will occur due to a spring at $x = 0$ with spring constant, k, and a sphere with lump mass, m, in one medium. Solve the PDE using diamond rule. (30%)

$$u_{tt} = c^2 u_{xx}, \quad for \ \ -\infty < x < \infty, \ \ t > 0$$

with initial condition of displacement

$$u(x,0) = \begin{cases} f(x), & for \ \ x > 0 \\ 0, & for \ \ x < 0 \end{cases}$$

with initial condition of velocity

$$u_t(x,0) = \begin{cases} 0, & for \ \ x > 0 \\ 0, & for \ \ x < 0 \end{cases}$$

$u(x,t)$ is continuous across $x = 0$,

$$u(0^+,t) = u(0^-,t)$$

Force can be transmitted across $x = 0$,

$$m\ddot{u}(0,t) + ku(0,t) = \rho c^2 u_x(0^+,t) - \rho c^2 u_x(0^-,t)$$

(1). Determine the solution in each region.
(2). Determine the ratio of transmission and reflection.
(3). Reduce the solution by $m \to 0$ and check the following solution if $k \to 0$

$$r(t) = u(0,t) = \int_0^t \frac{\rho c}{m} e^{-\frac{\rho c}{m}(t-\tau)} f(c\tau) d\tau$$

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