

工程數學 (四) - 偏微分方程

期中考 10:20-12:10, Apr.24, 1995 (OPEN BOOK)

I. Solve the PDE (20%)

$$u_{tt} = \begin{cases} 4u_{xx}, & \text{for } x < 0, t > 0 \\ 1u_{xx}, & \text{for } x > 0, t > 0 \end{cases}$$

with initial conditions

$$u(x, 0) = 0, u_t(x, 0) = 0$$

and $u(x, t)$ is continuous across $x = 0$, while

$$u_x(0^+, t) - u_x(0^-, t) = a \sin(\omega t)$$

where a, ω are two constants.

II. Reflection and transmission due to lump mass, m , in one medium using diamond rule. Solve the PDE (20%)

$$u_{tt} = c_1^2 u_{xx}, \quad \text{for } -\infty < x < \infty, t > 0$$

with initial condition of displacement

$$u(x, 0) = \begin{cases} f(x), & \text{for } x > 0 \\ 0, & \text{for } x < 0 \end{cases}$$

with initial condition of velocity

$$u_t(x, 0) = \begin{cases} 0, & \text{for } x > 0 \\ 0, & \text{for } x < 0 \end{cases}$$

$u(x, t)$ is continuous across $x = 0$,

$$u(0^+, t) = u(0^-, t)$$

Force can be transmitted across $x = 0$,

$$m\ddot{u}(t) = \rho c_1^2 u_x(0^+, t) - \rho c_1^2 u_x(0^-, t)$$

(1). Determine the ratio of transmission and reflection.

III. Solve the PDE (30%)

$$u_{tt} = c^2 u_{xx}, \quad \text{for } -\infty < x < \infty, t > 0$$

with initial conditions

$$u(x, 0) = 0, \dot{u}(x, 0) = \frac{1}{a}[H(x - a) - H(x + a)]$$

(1). As $a = 1$, check the same problem of homework.

(2). Discuss the limiting case for $a \rightarrow 0$.

IV. Solve the PDE (30%)

$$u_{tt} = u_{xx}, \quad \text{for } 0 < x < 1, t > 0$$

with initial conditions

$$u(x, 0) = \sin(5\pi x) + 0.5\sin(7\pi x), \dot{u}(x, 0) = 0$$

with boundary conditions

$$u(0, t) = u(1, t) = 0$$

V. What are PDE, diamond rule, characteristic and wave equation? (20%)

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