Solution stages using $p_0(s)$ and $q_0(s)$

1. Cauchy data $x_0(s), y_0(s)$ and $z_0(s)$
2. To determine $p_0(s)$ and $q_0(s)$,

$$F(x(s), y(s), u(s), p(s), q(s)) = 0$$

$$u'(s) = u_x x'(s) + u_y y'(s) = p(s)x'(s) + q(s)y'(s)$$

Two equations, two unknowns $p_0(s)$ and $q_0(s)$.

3. Solution steps:

Stage I: Governing equation and Cauchy data

$$F(x, y, u, u_x, u_y) = 0, u(x(s), y(s)) = u(s)$$

Stage II: Solvability condition:

$$F_p y'(s) - F_q x'(s) \neq 0$$

Stage III: Find $p_0(s)$ and $q_0(s)$,

$$F(x(s), y(s), u(s), p(s), q(s) = 0$$

$$u'(s) = u_x x'(s) + u_y y'(s) = p(s)x'(s) + q(s)y'(s)$$

Stage IV: Solve system ODEs

$$\frac{dx}{dt} = F_p, x(0, s) = x_0(s) \quad (1)$$

$$\frac{dy}{dt} = F_q, x(0, s) = y_0(s) \quad (2)$$

$$\frac{du}{dt} = pF_p + qF_q, u(0, s) = u_0(s) \quad (3)$$

$$\frac{dp}{dt} = -F_x - F_u p, p(0, s) = p_0(s) \quad (4)$$

$$\frac{dq}{dt} = -F_y - F_u q, q(0, s) = q_0(s) \quad (5)$$

$$(6)$$

Stage V: Solve

$$x(t, s)$$

$$y(t, s)$$

$$z(t, s)$$

$$p(t, s)$$

$$q(t, s)$$