1.(20%) For the non-exact differential equation $(x^2 + y^2)dx + xydy = 0$, can you find its general solution by (a) reducing it to a separable differential equation through homogeneous function method, (b) using integrating factor method, (c) using the linear differential equation method with variable transformation $u = y^2$, and (d) using the Bernoulli equation method.

2.(20%) Consider a single mass one-dimensional mechanical oscillator subjected to harmonic loading with the following equation of motion:

$$m\ddot{x}(t) + kx(t) = p_0 \sin \omega_d t, \quad x(0) = 0, \quad \dot{x}(0) = 0. \tag{1}$$

Solve the above initial-value problem for resonant case, i.e. $\omega = \sqrt{k/m} = \omega_d$, with any two (you can select) of the following methods: (a) undetermined coefficients, (b) variation of parameters, (c) inverse operator, and (d) the Laplace transform.

3.(20%) For the circular helix curve $\mathbf{r}(t) = x(t)\mathbf{i}+y(t)\mathbf{j}+z(t)\mathbf{k} = a\cos t\mathbf{i}+a\sin t\mathbf{j}+bt\mathbf{k},\ t \geq 0$, in $\mathbb{R}^3$, find (a) the arc length $s(t)$, and in terms of s the (b) unit tangent vector $\mathbf{T}$, (c) unit normal vector $\mathbf{N}$, (d) unit binormal vector $\mathbf{B}$, (e) the curvature κ and the torsion τ.

4.(20%) Evaluate the following contour integrals by using Green's Theorem:
(a) $\oint_C [(y - x^2e^x)dx + (\cos(2y^2) - x)dy]$,
where C is the rectangle with vertices at $(0, 1)$, $(1, 1)$, $(1, 3)$, and $(0, 3)$.
(b) $\oint_C [2x\cos(2y)dx - 2x^2 \sin(2y)dy]$,
with C any positively oriented closed path in the plane.

5.(20%) By using the Fourier transform method can you solve the following one-dimensional diffusion equation:

$$\begin{aligned}
&\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \delta(x)\delta(t),\\
&u(x, 0) = \delta(x),\\
&\lim_{|x|\to\infty} u(x, t) = 0.
\end{aligned}$$