1. Fill the number $(1, 2, 3 \cdots)$ in Table 1 for the corresponding trajectories.

Table 1: ODE Solutions on the phase plane (15 %)

system	(a)	(b)	(c)	(d)	(e)
phase plane					

(註: 請將上表抄入答案紙才計分)

System (a).

$$\dot{x} = y + x - x(x^2 + y^2)$$

$$\dot{y} = -x + y - y(x^2 + y^2)$$

System (b).

$$\dot{x} = y$$

$$\dot{y} = -sin(x)$$

System (c).

$$\ddot{x} + \frac{x}{|x|} = 0$$

System (d). $\eta = 2.0, \omega = 1.0$,

$$\ddot{x} + \omega^2 \eta \frac{|x|}{|\dot{x}|} \dot{x} + \omega^2 x = 0$$

System (e).

$$\dot{x} = x$$

$$\dot{y} = y$$

where $|x|$ denotes the absolute value of x.

2. Given the following ordinary differential equation (ODE):

$$\ddot{x}(t) + \frac{x(t)}{|x(t)|} = 0$$

subjected to $x(0) = 0, \dot{x}(0) = 1$. Answer the following questions.
  (a). Is this ODE linear or nonlinear ? (5 %)
  (b). Plot the trajectory in the phase plane $(x, \dot{x})$. (5 %)
  (c). Is the solution periodic ? If yes, what is the period ? (5 %)

3. Given the following partial differential equations:

$$\nabla^2 u(\mathbf{x}) = 0, \ \mathbf{x} \in \ D \tag{1}$$

$$\nabla^2 u(\mathbf{x}) + 5u(\mathbf{x}) = 0, \ \mathbf{x} \in \ D \tag{2}$$

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial^2 u(x,t)}{\partial t^2} \tag{3}$$

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial u(x,t)}{\partial t} \tag{4}$$

Please fill the number (1,2,3,4) into Table 2. (10 %)

Table 2: PDE mapping

Eqs. type	Laplace eq.	Helmholtz eq.	Wave eq.	Heat eq.	Diffusion eq.
Eqs. No.					

(註: 請將上表抄入答案紙才計分)

4. Solve the root of complex equation for z which satifies

$$\sqrt{z} = -i$$

where $i^2 = -1$. Find the complex value, z. (5 %) After you obtain the solution, please check it by substituting into $\sqrt{z} = -i$, what happen ? Give your comments. (5 %)

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存檔:$math99.ctx$建檔Apr./18/'99