Half-circle Laplace problem

<u>Trefftz method</u>

猜基底函數： $u(r,q)=a+b\ln r+\sum_{n=1}^{\infty}(a_n r^n\cos nq+b_n r^n\sin nq)+\sum_{n=1}^{\infty}(c_n\frac{1}{r^n}\cos nq+d_n\frac{1}{r^n}\sin nq)$

決定基底函數： $u(r,q)=\sum_{n=1}^{q}r^n b_n\sin nq$ （**內域**、no singularity、B.C.、**反對稱**）

$u_0 q(p-q)=\sum_{n=1}^{q}r^n b_n\sin nq$ （Fourier half-sine expansion）

$b_n=\frac{2}{p}\int_0^p u_0 q(p-q)\sin nq\,dq=u_0[\frac{4-4(-1)^n}{n^3 p}]$

$u(r,q)=\sum_{n=1}^{\infty}u_0 r^n[\frac{4-4(-1)^n}{n^3 p}]\sin nq$

<u>Boundary Integral Equations</u>

積分方程	$2pu(x)=\int_B[T(s,x)u(s)-U(s,x)t(s)]dB(s)$	$0=\int_B[T(s,x)u(s)-U(s,x)t(s)]dB(s)$ **零場積分方程**
退化核	$U(s,x)=\begin{cases}\ln R-\sum_{m=1}^{\infty}\frac{1}{m}(\frac{r}{R})^m\cos m(q-f), & R>r\\ \ln r-\sum_{m=1}^{\infty}\frac{1}{m}(\frac{R}{r})^m\cos m(q-f), & r>R\end{cases}$	$T(s,x)=\frac{U(s,x)}{\partial n_s}=\begin{cases}\frac{1}{R}+\sum_{m=1}^{\infty}(\frac{r^m}{R^{m+1}})\cos m(q-f), & R>r\\ -\sum_{m=1}^{\infty}(\frac{R^{m-1}}{r^m})\cos m(q-f), & r>R\end{cases}$
邊界條件	$u(s)=\begin{cases}u_0 q(p-q), & 0<q<p\\ u_0(2p-q)(p-q), & p<q<2p\end{cases}$	$t(s)=p_0+\sum_{n=1}^{\infty}(p_n\cos nq+q_n\sin nq)$ **未知邊界密度**

零場積分方程式：

$$0=\int_B[T(s,x)u(s)-U(s,x)t(s)]dB(s)$$

$$=\int_0^p[-\sum_{m=1}^{\infty}(\frac{R^{m-1}}{r^m})\cos m(q-f)][u_0 q(p-q)]a\,dq+\int_p^{2p}[-\sum_{m=1}^{\infty}(\frac{R^{m-1}}{r^m})\cos m(q-f)][u_0(2p-q)(p-q)]a\,dq$$

$$-\int_0^{2p}[\ln r-\sum_{m=1}^{\infty}\frac{1}{m}(\frac{R}{r})^m\cos m(q-f)][p_0+\sum_{n=1}^{\infty}(p_n\cos nq+q_n\sin nq)]a\,dq$$

$$=\sum_{m=1}^{\infty}au_0(\frac{R^{m-1}}{r^m})(\frac{4\cos mp-4}{m^3})\sin mf-2pap_0\ln r+\sum_{m=1}^{\infty}au_0(\frac{R}{r})^m[p_m(\frac{p}{m}\cos mf)+q_n(\frac{p}{m}\sin nf)]$$

$p_0 = 0$、 $p_m = 0$、 $q_m = \dfrac{4 - 4\cos mp}{Rm^2 p}$

$$t(s) = \sum_{n=1}^{\infty} \left(\frac{4 - 4\cos np}{Rn^2 p}\right) \sin nq$$

域內點積分方程式：

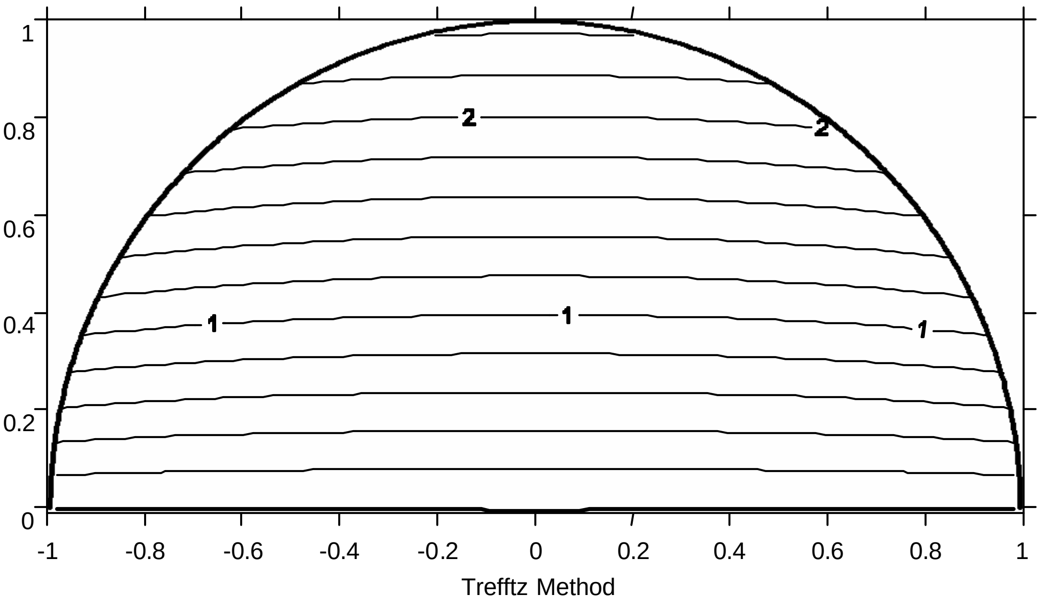
$$2pu(x) = \int_B [T(s,x)u(s) - U(s,x)\, t(s)]dB(s)$$

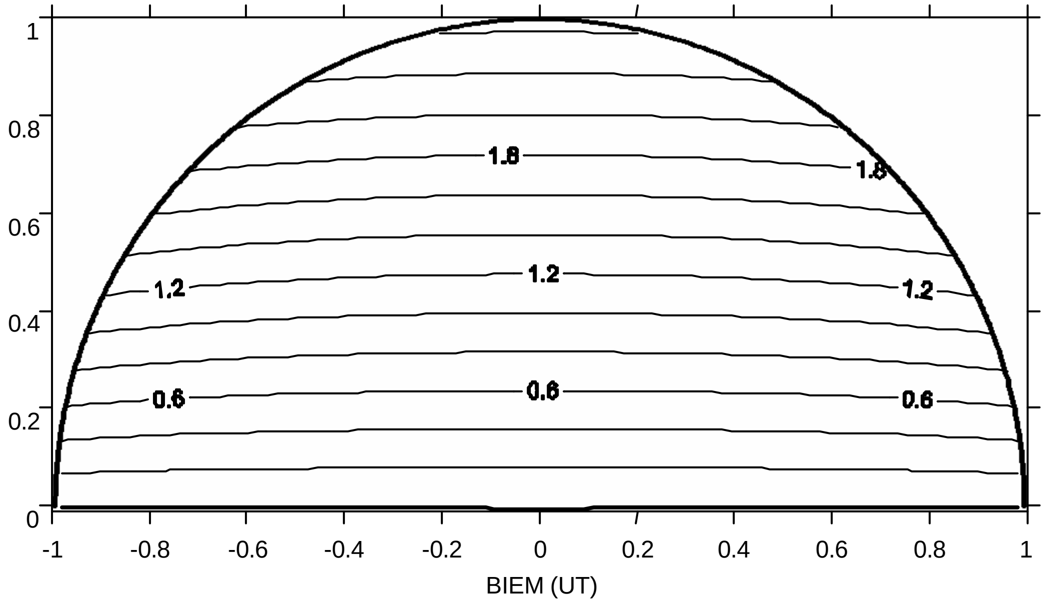
$$= \int_0^p [\frac{1}{R} + \sum_{m=1}^{\infty} (\frac{r^m}{R^{m+1}}) \cos m(q - f)][u_0 q(p - q)]adq + \int_p^{2p} [\frac{1}{R} + \sum_{m=1}^{\infty} (\frac{r^m}{R^{m+1}}) \cos m(q - f)][u_0 (2p - q)(p - q)]adq$$

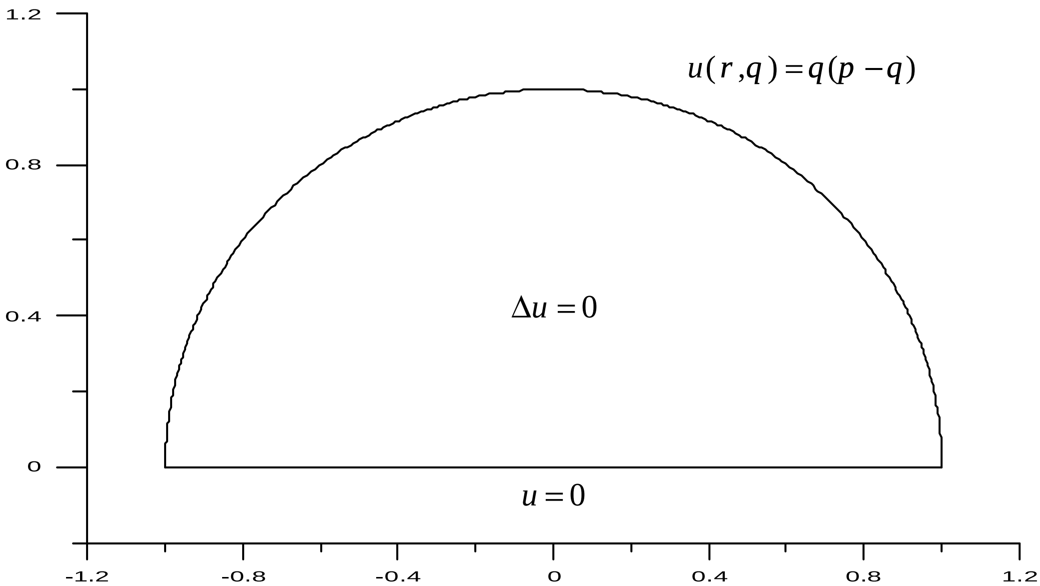
$$- \int_0^{2p} [\ln R - \sum_{m=1}^{\infty} \frac{1}{m} (\frac{r}{R})^m \cos m(q - f)][\sum_{n=1}^{\infty} (\frac{4 - 4\cos np}{Rn^2 p}) \sin nq\,]adq$$

$$= \sum_{m=1}^{\infty} au_0 (\frac{r^m}{R^{m+1}})[(\frac{8 - 8\cos mp}{m^3}) \sin mf\,]$$

$$u(x) = \frac{1}{2p} \sum_{m=1}^{\infty} au_0 (\frac{r^m}{R^{m+1}})[(\frac{8 - 8\cos mp}{m^3}) \sin mf\,]$$







評論：

(1)Trefftz Method **猜基底要有技巧。**

(2)**積分方程法較有系統。**

(3)**兩者所得到的解析解相同。**

陳正宗教授指導 沈文成製表 half04.doc