國立臺灣海洋大學九十四學年度研究所碩士班暨碩士在職專班考試試題

系所名稱：河海工程學系(大地工程組)

科目名稱：工程數學　　＊使用計算機

1. 答案以橫式由左至右書寫。2.請依題號順序作答。

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1. (1) $\nabla \cdot \underset{\sim}{r} = ?$ where $\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$.(2%)

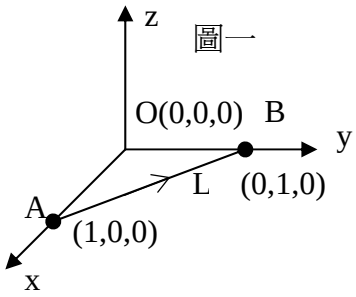
(2) Line integral $\oint_C \underset{\sim}{r} \cdot \underset{\sim}{n}\, ds = ?$

where C is the closed loop of OAB.(圖一) (4%)

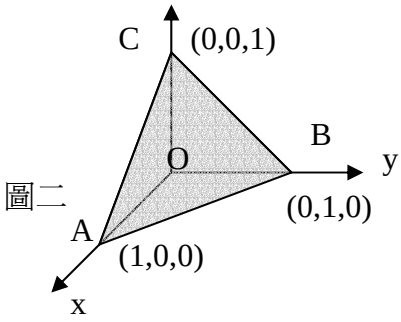
(3) Surface integral: $\iint_S \underset{\sim}{r} \cdot \underset{\sim}{n}\, dS = ?$

where S is the surface of plane ABC.(圖二) (4%)

(Note that $\underset{\sim}{n}$ is the normal vectors of ds and dS, respectively)



圖一



圖二

2. Give a function $y(x)$ with a period 2 and
$y(x) = 0, -1 < x < 0$ and $y(x) = 1, 0 < x < 1$

(1) Decompose the function into even function of $y_e(x)$ and odd function of $y_o(x)$ (2%)

(2) Plot $y(x)$, $y_e(x)$ and $y_o(x)$. (3%)

(3) Expand $y_e(x)$ and $y_o(x)$ into Fourier series. (5%)

(4) Is termwise (term by term) differentiation legal with respect to any Fourier series ? (5%)

3. Complex variable

(1) $\oint_C \frac{1}{z} dz = ?$ where C is the unit circle in a counterclockwise direction. (2%)

(2) What is the definition of Cauchy principal value (CPV) ? (3%)

(3). $CPV \int_{-\infty}^{\infty} \frac{\cos(mx)}{x-a} dx = ?, \text{for } a \text{ real}, m > 0$ (4%)

(4). $CPV \int_{-\infty}^{\infty} \frac{\sin(mx)}{x-a} dx = ?, \text{ for } a \text{ real}, m > 0$ (4%)

(5). What is Hilbert transform ? (2%)

4. Solve the following partial differential equation.

$yu_x - xu_y = 3x$ subject to $u(x,0) = x^2$ Solve $u(x,y) = ?$ (10%)

5. 請問一階、線性、非齊次向量微分方程式之定義爲何？(10%)

6. 利用矩陣方法解系統 (10%)

$\dot{x} = x + 2y$

$\dot{y} = 4x + 3y$

7. 證明 Cayley-Hamilton 定理對下列矩陣成立 (10%)

$$\begin{bmatrix} 4 & 1 & 0 \\ -1 & 2 & 0 \\ 2 & 1 & -3 \end{bmatrix}$$

8. 求 $f(x) = e^x$ 在 $\{0, \pi\}$ 內之 Fourier Cosine 級數。(10%)

9. 利用卷積(convolution) 求逆 Laplace 轉換 (10%)

$$L^{-1}\{\frac{1}{s(s^2+1)}\}$$