國立臺灣海洋大學九十四學年度研究所碩士班暨碩士在職專班考試試題

系所名稱：河海工程學系(海洋工程組、水資源與環境工程組)

科目名稱：工程數學　　＊使用計算機

1. 答案以橫式由左至右書寫。2.請依題號順序作答。

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1. (1) $\nabla \cdot \underset{\sim}{r} = ?$ where $\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$.(2%)

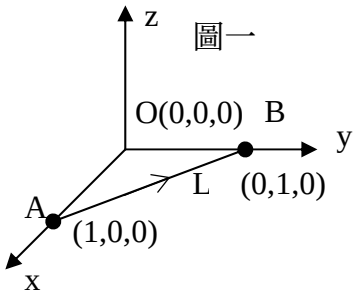
(2) Line integral $\oint_C \underset{\sim}{r} \cdot \underset{\sim}{n}\, ds = ?$

where C is the closed loop of OAB.(圖一) (4%)

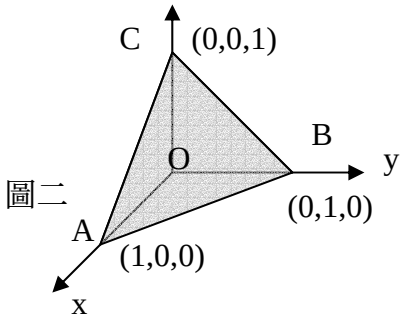
(3) Surface integral: $\iint_S \underset{\sim}{r} \cdot \underset{\sim}{n}\, dS = ?$

where S is the surface of plane ABC.(圖二) (4%)

(Note that $\underset{\sim}{n}$ is the normal vectors of ds and dS, respectively)



圖一



圖二

2. Give a function $y(x)$ with a period 2 and

$y(x) = 0, -1 < x < 0$ and $y(x) = 1, 0 < x < 1$

(1) Decompose the function into even function of $y_e(x)$ and odd function of $y_o(x)$ (2%)

(2) Plot $y(x)$, $y_e(x)$ and $y_o(x)$. (3%)

(3) Expand $y_e(x)$ and $y_o(x)$ into Fourier series. (5%)

(4) Is termwise (term by term) differentiation legal with respect to any Fourier series ? (5%)

3. Complex variable

(1) $\oint_C \frac{1}{z} dz = ?$ where C is the unit circle in a counterclockwise direction. (2%)

(2) What is the definition of Cauchy principal value (CPV) ? (3%)

(3). $CPV \int_{-\infty}^{\infty} \frac{\cos(mx)}{x-a} dx = ?$, for a real, $m > 0$ (4%)

(4). $CPV \int_{-\infty}^{\infty} \frac{\sin(mx)}{x-a} dx = ?$, for a real, $m > 0$ (4%)

(5). What is Hilbert transform ? (2%)

4. Solve the following partial differential equation.

$yu_x - xu_y = 3x$ subject to $u(x,0) = x^2$ Solve $u(x,y) = ?$ (10%)

5. Find the general solution, y(x), of the following equations: (20%)

(1) $By'(x)+Gy(x)+Ky''(x)=0$, where B, G, K are constants

(2) $y''+2y'+3y=2+x$

(3) $2y+3xy'+x^2y''=0$, $y(1)=1$, $y'(1)=1$

6. $\begin{cases} X_1-2X_2+X_3=2 \\ 5X_1+X_2-X_3=3 \\ X_1+2X_2+X_3=5 \end{cases}$ (15%)

(1) Write the system of equations in matrix form AX=B

(2) Calculate the eigen-values and eigen-vectors of matrix A

(3) Find the Inverse matrix A^{-1}

(4) Solve $X=A^{-1}B$

7. $f(t)=\begin{cases} Cos(t) & for \quad 0\le t<2\pi \\ 0 & for \quad t<0 \ and \ for \ t\ge 2\pi \end{cases}$ (15%)

$H(t-a)=\begin{cases} 1 & for \quad t\ge a \\ 0 & for \quad t<a \end{cases}$

(1) Plot the figure [f(t) vs. t] and expressed f(t) in terms of the Heaviside function H(t)

(2) Calculate the Laplace Transform of f(t)

(3) Calculate the Fourier Transform of f(t)