# Part III

# Plate vibration

# Outlines

1. Introduction
2. Plate vibration
3. Derivation by Circulants
4. SVD updating terms
5. Conclusions

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# Vibration of plates

## Governing Equation:

$$\nabla^4 u(x) = \lambda^4 u(x),\ x \in \Omega$$

$$\lambda^4 = \frac{\omega^2 \rho h}{D}$$

$$D = \frac{E h^3}{12(1-\nu^2)}$$

$\omega$ is the angle frequency

$\rho$ is the surface density

$D$ is the flexural rigidity

$h$ is the plates thickness

$E$ is the Young's modulus

$\nu$ is the Poisson ration

armonic operator

eral displacement

equency parameter

main of the thin plates

# Field representation using RBF

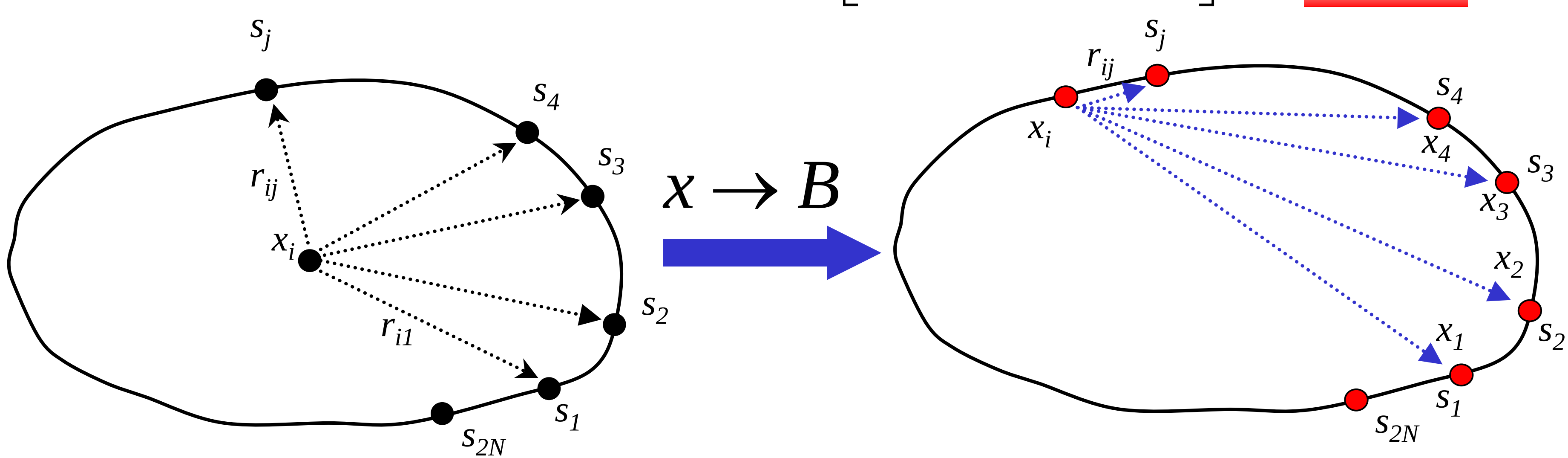
$$u(x) = \sum c_j \psi(x_i, s_j)$$

$$\psi(x, s) = \psi(r)$$

$$\{u\} = \begin{bmatrix} \ddots & * & * & * & * \\ * & \ddots & * & * & * \\ * & * & \ddots & * & * \\ * & * & * & \ddots & * \\ * & * & * & * & \ddots \end{bmatrix} \{c\}$$

$r \neq 0$

$r = 0$

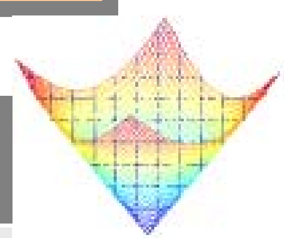


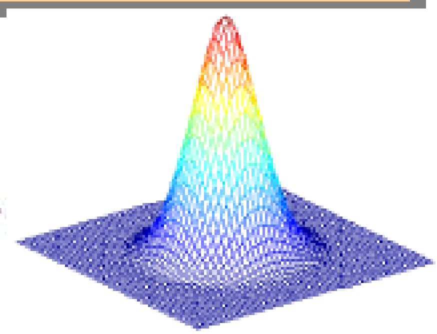

**Field representation** **To match B.C.**

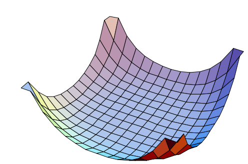
# Data bank of RBF

**Radial basis function (RBFs)**

- **Mesh method**
  - **Globally-supported RBFs**
    - **DRBEM** — Nardini, Brebbia, 1982 → $\psi(r) = 1 + r$
    - **Method of particular integral** — Ahmad & Banerjee, 1986 → $\psi(r) = C - r$
      Method of particular integral Ahmad & Banerjee 1986
    - → **Equivalence** — Polyzos & Beskos, 1994
- **Meshless method**
  - **Compactly-supported RBFs** — C.S. Chen → $\psi(r) = (1-r)_+^4(4r+1)$
  - **Globally-supported RBFs**
    - **Volume potential** → **Method of fundamental solution** →
      $$\psi(r) = U_c(s,x) = U_c(|x-s|) = U_c(r)$$
      (Potential theory)
    - **The present method** → **Imaginary-part fundamental solution** →
      $$\psi(r) = J_0(\lambda r) + I_0(\lambda r)$$







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# Displacement field of plate vibration

$$u(s,x)=\sum_{j=1}^{2N}P(s_j,x)\phi_j+\sum_{j=1}^{2N}Q(s_j,x)\psi_j$$

**$2N$ is the number of boundary nodes**

***s* is the source point**

***x* is the collocation point**

**$f_j$ and $y_j$ are the unknown densities**

***P* and *Q* can be obtained from either two combinations of *U*, *Q*, *M* and *V***

# Four kernels

## Imaginary-part fundamental solution

$$U(s,x) = \mathrm{Im}\{\frac{i}{8\lambda^2}(H_0^{(2)}(\lambda r) + H_0^{(1)}(i\lambda r))\}$$

$$U(s,x) = \frac{1}{8\lambda^2}(J_0(\lambda r) + I_0(\lambda r))$$

# Four kernels

$$\Theta(s,x) = K_{\theta}(U(s,x))$$

$$M(s,x) = K_{m}(U(s,x))$$

$$V(s,x) = K_{v}(U(s,x))$$

# Operators

$$K_\theta(\cdot) = \frac{\partial(\cdot)}{\partial n}$$

$$K_m(\cdot) = \nu \nabla^2(\cdot) + (1-\nu)\frac{\partial^2(\cdot)}{\partial n^2}$$

$$K_v(\cdot) = \frac{\partial \nabla^2(\cdot)}{\partial n} + (1-\nu)\frac{\partial}{\partial t}\left(\frac{\partial^2(\cdot)}{\partial n \partial t}\right)$$

# Slope, Moment and Shear

**Slope** $$\theta(x) = K_{\theta}(u(x))$$

**Moment** $$m(x) = K_{m}(u(x))$$

**Shear** $$v(x) = K_{v}(u(x))$$

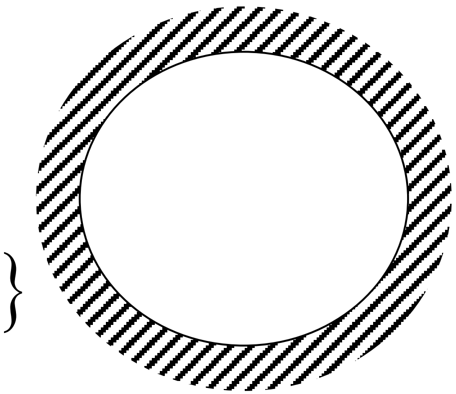
# True eigenequation of three cases

| | B.C. | True eigenequation |
|---|---|---|
| Clamped plate | $u(x)=0$<br>$q(x)=0$ | $J'_\ell(\lambda\rho)I_\ell(\lambda\rho)-I'_\ell(\lambda\rho)J_\ell(\lambda\rho)=0$ |
| Simply-supported plate | $u(x)=0$<br>$m(x)=0$ | $\frac{I_{\ell+1}(\lambda\rho)}{I_\ell(\lambda\rho)}+\frac{J_{\ell+1}(\lambda\rho)}{J_\ell(\lambda\rho)}=\frac{2\lambda\rho}{(1-\nu)}$ |
| Free plate | $m(x)=0$<br>$v(x)=0$ | $(\ell^2(\ell^2-1)(-1+\nu)^2+\lambda^4 a^4)(J_{\ell+1}(\lambda a)I_\ell(\lambda a)+J_\ell(\lambda a)I_{\ell+1}(\lambda a))$<br>$+2\ell\lambda^2 a^2(1-\ell)(-1+\nu)(J_{\ell+1}(\lambda a)I_\ell(\lambda a)-J_\ell(\lambda a)I_{\ell+1}(\lambda a))$<br>$+\lambda a(-1+\nu)(2\lambda^2 a^2 J_{\ell+1}(\lambda a)I_{\ell+1}(\lambda a)+4\ell^2(-1+\ell)J_\ell(\lambda a)I_\ell(\lambda a)=0$ |

# Clamped plate

$$u(x) = 0 \Rightarrow [U]\{\phi\} + [\Theta]\{\psi\} = \{0\}$$

$$\theta(x) = 0 \Rightarrow [U_\theta]\{\phi\} + [\Theta_\theta]\{\psi\} = \{0\}$$



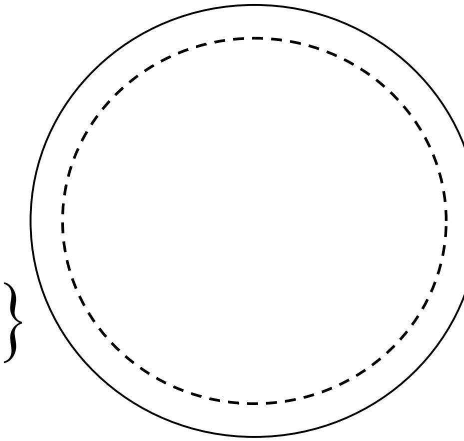
**Nontrivial**

$$0 = \det\left[SM^C\right] \Leftarrow \begin{bmatrix} U & \Theta \\ U_\theta & \Theta_\theta \end{bmatrix} \begin{Bmatrix} \phi \\ \psi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

# Simply-supported plate

$$u(x) = 0 \Rightarrow [U]\{\phi\} + [\Theta]\{\psi\} = \{0\}$$

$$m(x) = 0 \Rightarrow [U_m]\{\phi\} + [\Theta_m]\{\psi\} = \{0\}$$



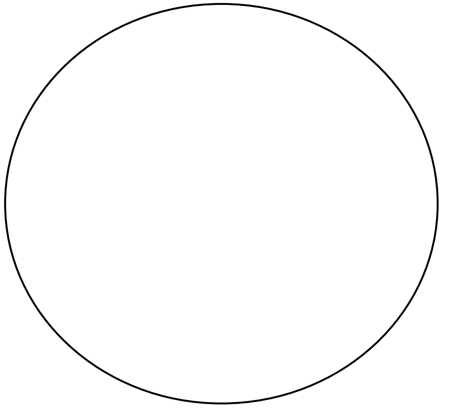
Nontrivial

$$0 = \left[ SM^S \right] \Leftarrow \begin{bmatrix} U & \Theta \\ U_m & \Theta_m \end{bmatrix} \begin{Bmatrix} \phi \\ \psi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

# Free plate

$$m(x) = 0 \Rightarrow [U_m]\{\phi\} + [\Theta_m]\{\psi\} = \{0\}$$

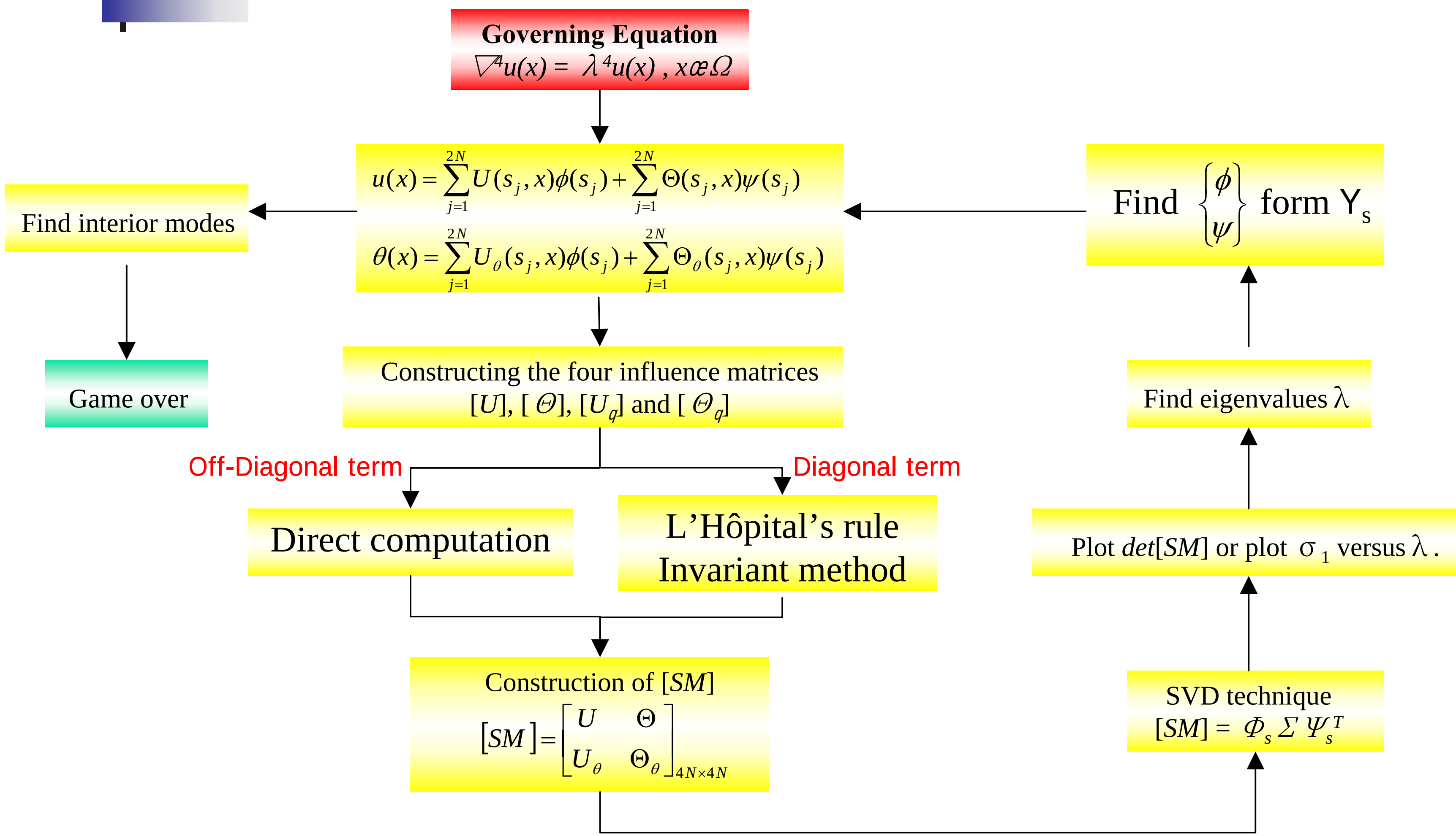
$$v(x) = 0 \Rightarrow [U_v]\{\phi\} + [\Theta_v]\{\psi\} = \{0\}$$



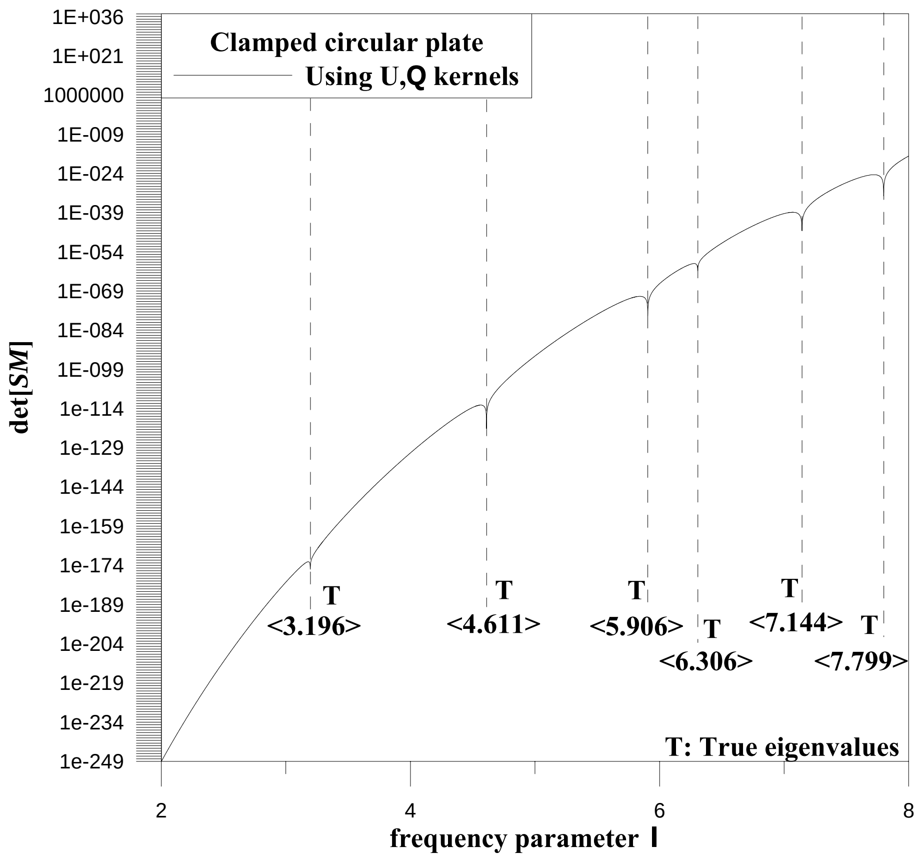
Nontrivial

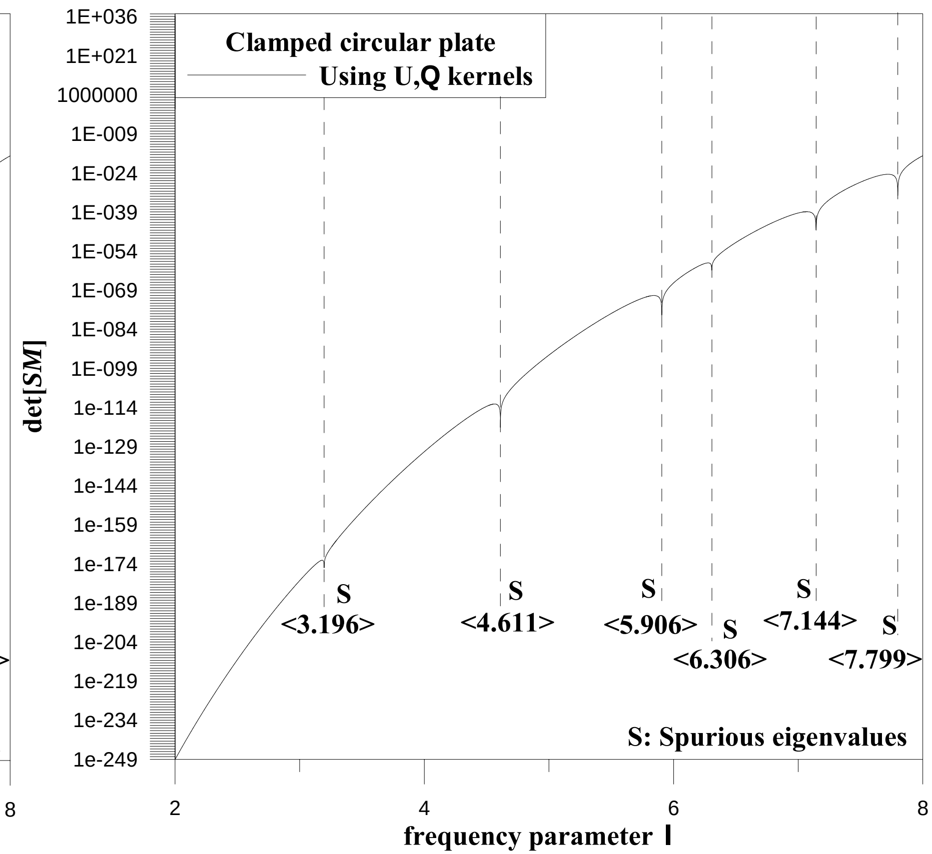
$$0 = \left[SM^{F}\right] \Leftarrow \begin{bmatrix} U_m & \Theta_m \\ U_v & \Theta_v \end{bmatrix} \begin{Bmatrix} \phi \\ \psi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

# Flow chart of the present method for clamped case by using U and Q kernels

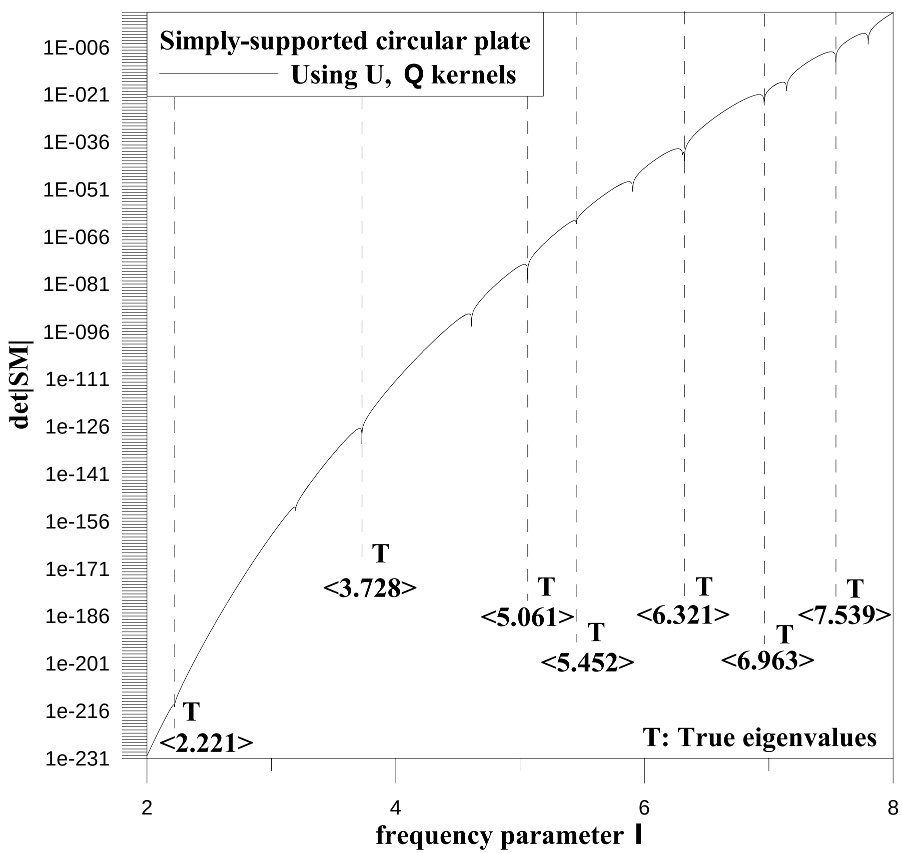


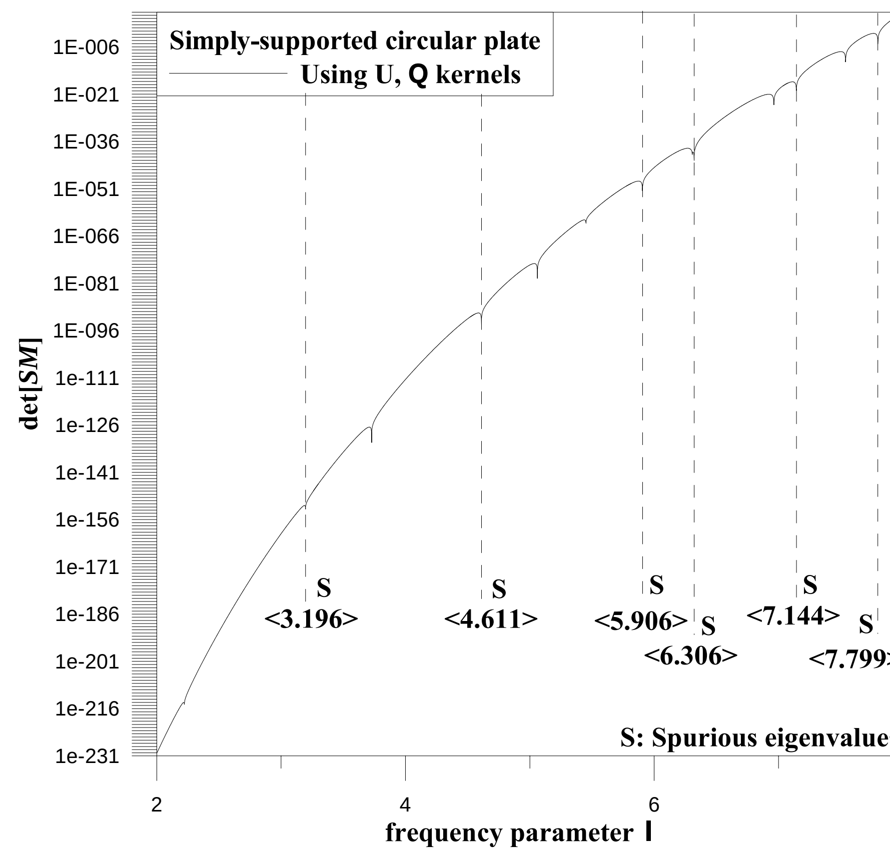
# Case 1: Circular clamped plate using the present method



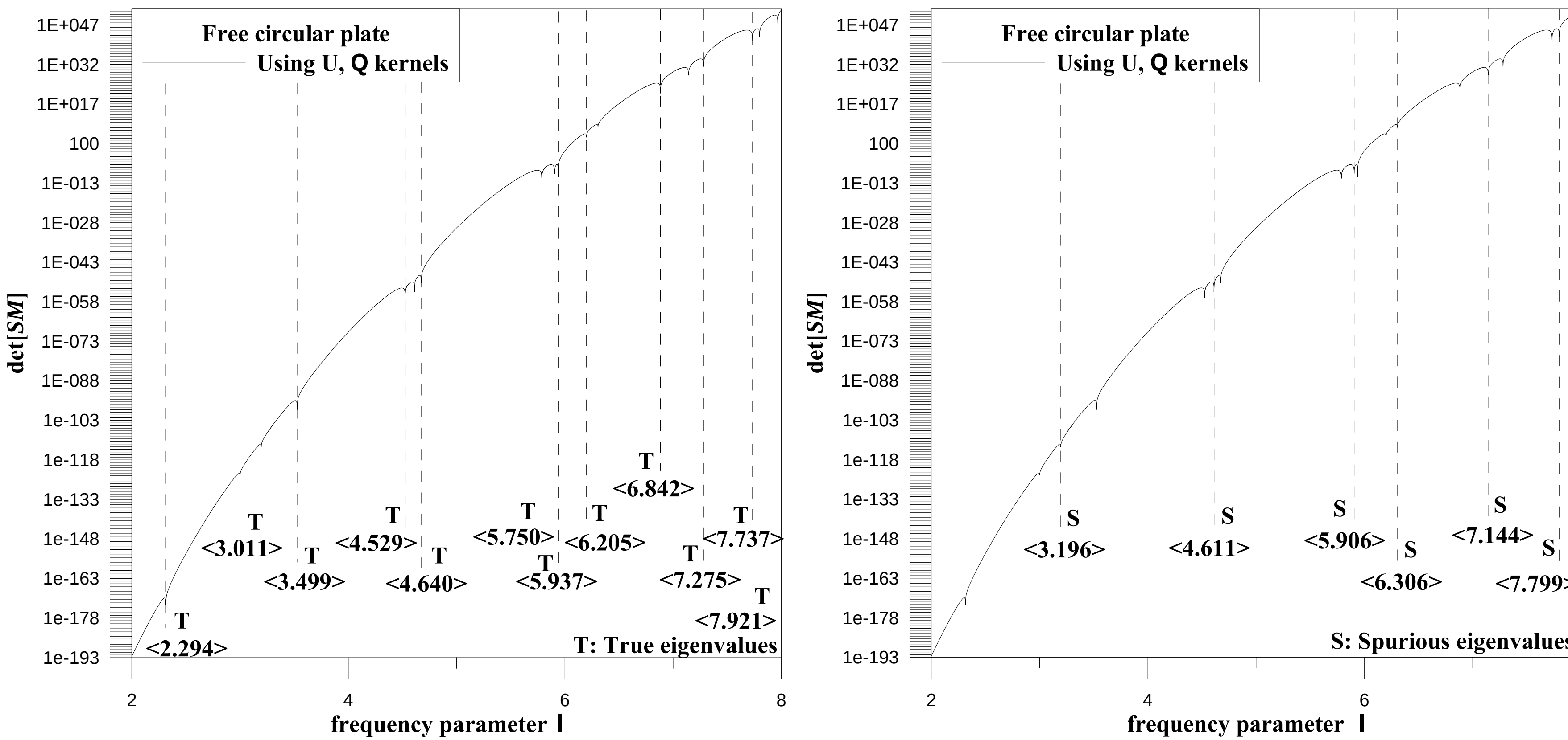



# Case 2: Circular simply-supported plate using the present method

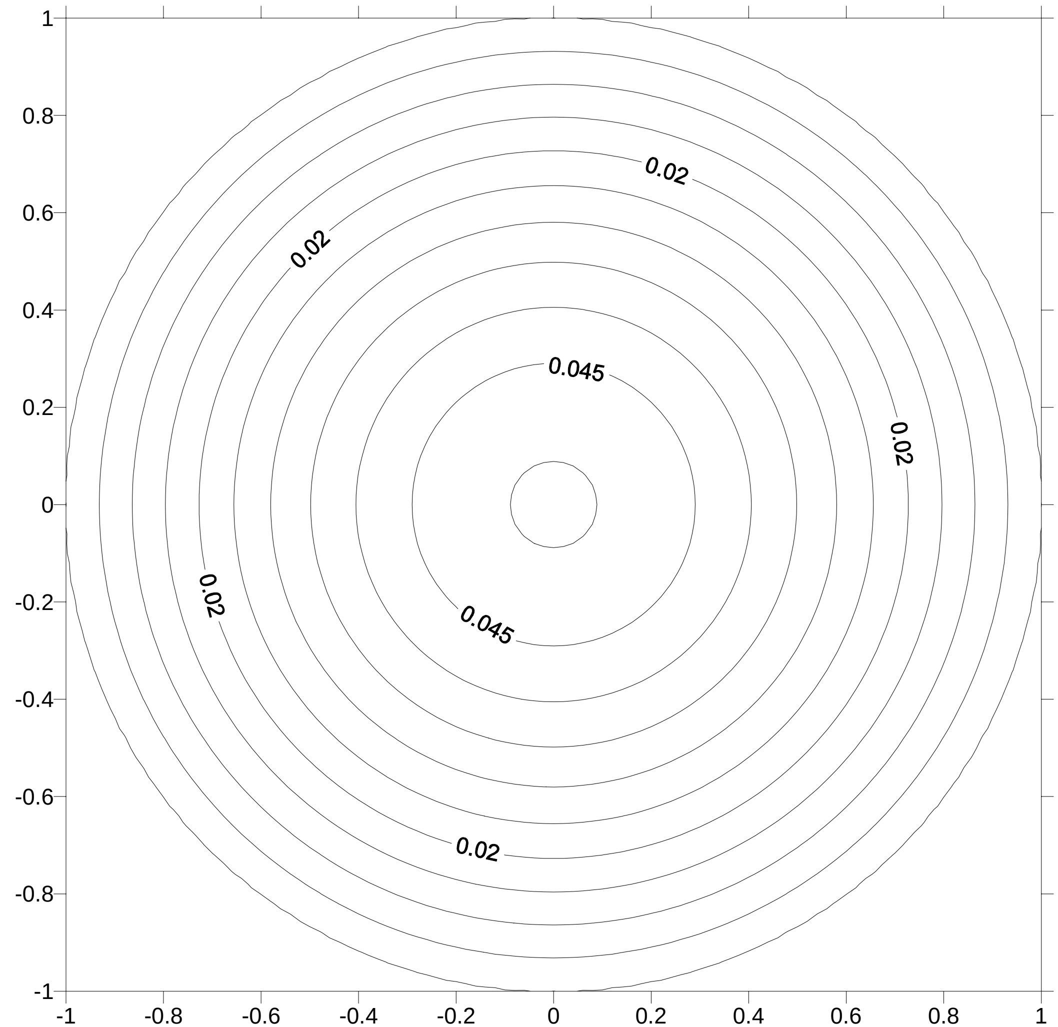


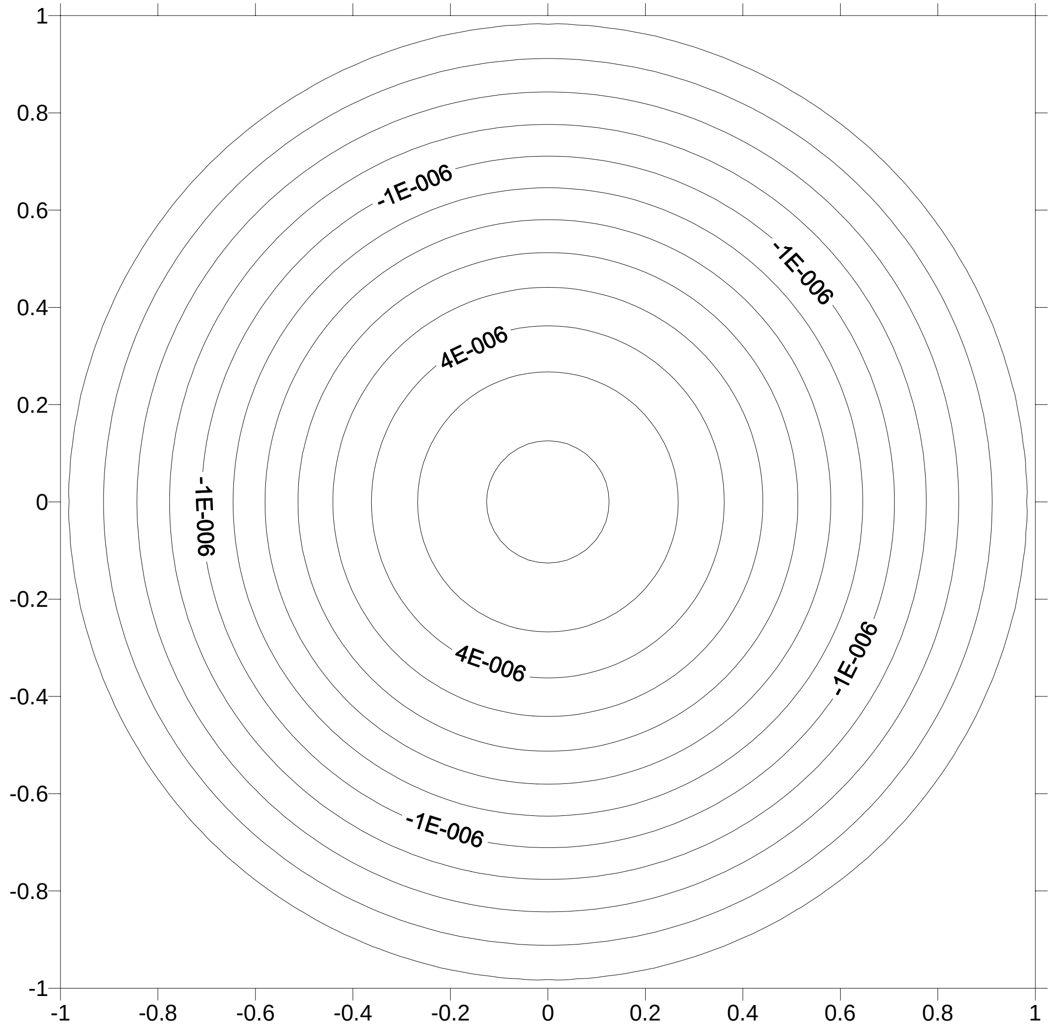
# Case 3: Circular free plate using the present method



# Mode 1 (λ =2.221)

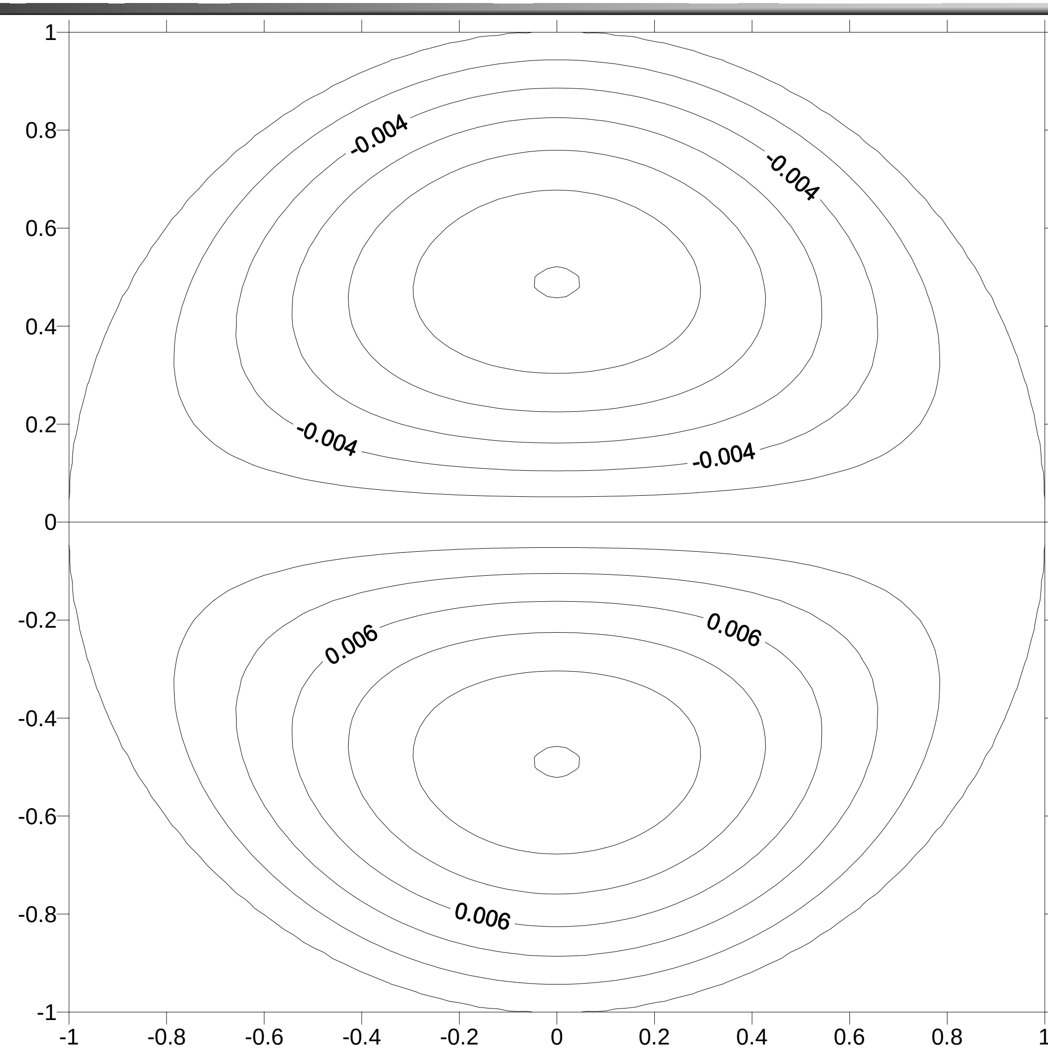

0.02
0.02
0.045
0.02
0.02
0.045
0.02
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

# Mode 2 (λ=3.196)



1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
-1E-006
-1E-006
4E-006
-1E-006
4E-006
-1E-006
-1E-006

# Mode 3 (λ =3.728)



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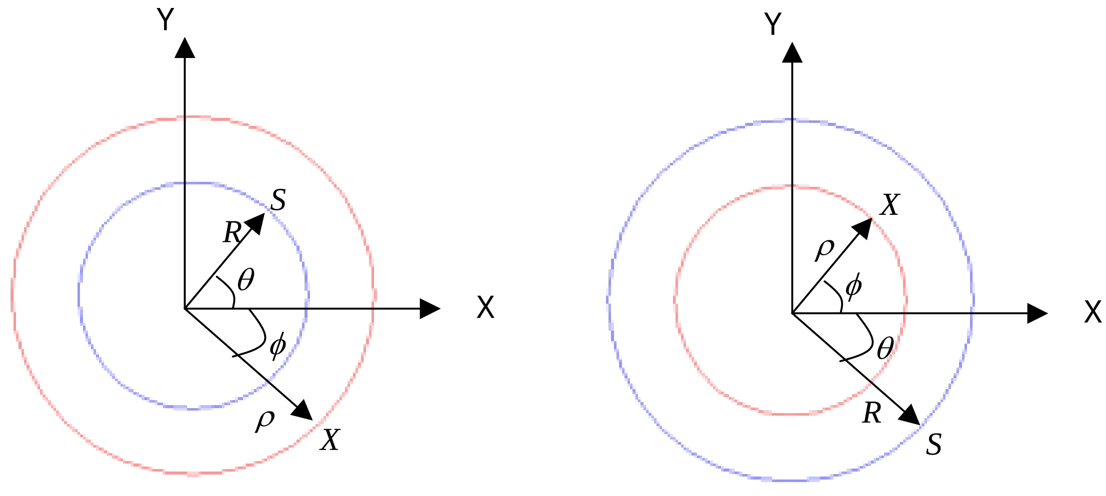
# Degenerate kernels for circular case

**The degenerate kernels for interior and exterior problems:**




Exterior problem

Interior problem

**Blue: field points**
**Red: source points**

$$U(s,x)=\begin{cases} U^{I}(R,\theta;\ \rho,\phi)=\dfrac{1}{8\lambda^2}\displaystyle\sum_{m=-\infty}^{\infty}[J_m(\lambda R)J_m(\lambda\rho)+(-1)^m I_m(\lambda R)I_m(\lambda\rho)](\cos(m(\theta-\phi))), & R>\rho \\ U^{E}(R,\theta;\ \rho,\phi)=\dfrac{1}{8\lambda^2}\displaystyle\sum_{m=-\infty}^{\infty}[J_m(\lambda\rho)J_m(\lambda R)+(-1)^m I_m(\lambda\rho)I_m(\lambda R)](\cos(m(\theta-\phi))), & R<\rho \end{cases}$$

# Circulants

$$[K]=\begin{bmatrix} a_0 & a_1 & a_2 & \cdots & a_{2N-2} & a_{2N-1} \\ a_{2N-1} & a_0 & a_1 & \cdots & a_{2N-3} & a_{2N-2} \\ a_{2N-2} & a_{2N-1} & a_0 & \cdots & a_{2N-4} & a_{2N-=3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_2 & a_3 & a_4 & \cdots & a_0 & a_1 \\ a_1 & a_2 & a_3 & \cdots & a_{2N-1} & a_0 \end{bmatrix}$$

$$a_{j-i} = K(s_j, x_i)$$

$$[K]=a_0 I + a_1 (C_{2N})^1 + a_2 (C_{2N})^2 + \cdots + a_{2N-1} (C_{2N})^{2N-1}$$

# Circulants

$$C_{2N} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}_{2N\times 2N}$$

$$\alpha^{2N} - 1 = 0$$

$$\alpha_{\ell} = e^{i\frac{2\pi\ell}{2N}} = \cos(\frac{2\pi\ell}{2N}) + i\sin(\frac{2\pi\ell}{2N})$$

# Circulants

$$\lambda_\ell^{[U]} = a_0 + a_1\alpha_\ell + a_2\alpha_\ell^2 + \cdots + a_{2N-1}\alpha_\ell^{2N-1}$$

$$\ell = 0, \pm 1, \pm 2, \cdots, \pm(N-1), N$$

$$\lambda_\ell^{[U]} = \frac{N}{4\lambda^2}[J_\ell(\lambda\rho)J_\ell(\lambda\rho) + (-1)^\ell I_\ell(\lambda\rho)I_\ell(\lambda\rho)]$$

# The eigenvalues of matrices

$$[U]=\Phi\Sigma_U\Phi^T$$

$$=\Phi\begin{bmatrix}\lambda_0^{[U]} & 0 & 0 & \cdots & 0 & 0 & 0\\ 0 & \lambda_1^{[U]} & 0 & \cdots & 0 & 0 & 0\\ 0 & 0 & \lambda_{-1}^{[U]} & \cdots & 0 & 0 & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & \cdots & \lambda_{(N-1)}^{[U]} & 0 & 0\\ 0 & 0 & 0 & \cdots & 0 & \lambda_{-(N-1)}^{[U]} & 0\\ 0 & 0 & 0 & \cdots & 0 & 0 & \lambda_N^{[U]}\end{bmatrix}_{2N\times 2N}\Phi^T$$

# Relationships

$$[U] \xrightarrow{SVD} \Sigma_U \rightarrow \lambda_\ell^{[U]}$$

$$[\Theta] \xrightarrow{SVD} \Sigma_\Theta \rightarrow \lambda_\ell^{[\Theta]}$$

$$[U_\theta] \xrightarrow{SVD} \Sigma_{U_\theta} \rightarrow \mu_\ell^{[U]}$$

$$[\Theta_\theta] \xrightarrow{SVD} \Sigma_{\Theta_\theta} \rightarrow \mu_\ell^{[\Theta]}$$

Diagonal matrix

Diagonal element

# Determinant (for clamped)

$$\left[SM^C\right]=\begin{bmatrix}\Phi\Sigma_U\Phi^T & \Phi\Sigma_\Theta\Phi^T \\ \Phi\Sigma_{U_\theta}\Phi^T & \Phi\Sigma_{\Theta_\theta}\Phi^T\end{bmatrix}_{2N\times 2N}$$

$$=\begin{bmatrix}\Phi & 0 \\ 0 & \Phi\end{bmatrix}\begin{bmatrix}\Sigma_U & \Sigma_\Theta \\ \Sigma_{U_\theta} & \Sigma_{\Theta_\theta}\end{bmatrix}\begin{bmatrix}\Phi & 0 \\ 0 & \Phi\end{bmatrix}^T$$

# Eigenequation for clamped boundary

$$\det[SM^C]$$

$$= \prod_{\ell=-(N+1)}^{N} (\lambda_\ell^{[U]}\mu_\ell^{[\Theta]} - \lambda_\ell^{[\Theta]}\mu_\ell^{[U]})$$

$$= \prod_{\ell=-(N+1)}^{N} \frac{(-1)^\ell N^2}{16\lambda^2}[J_{\ell+1}(\lambda\rho)I_\ell(\lambda\rho) + I_{\ell+1}(\lambda\rho)J_\ell(\lambda\rho)]$$

$$\{J_{\ell+1}(\lambda\rho)I_\ell(\lambda\rho) + I_{\ell+1}(\lambda\rho)J_\ell(\lambda\rho)\} = 0,$$

$$\ell = 0, \pm 1, \pm 2, \cdots, \pm (N-1), N$$

# Determinant (for simply-supported)

$$[SM^S] = \begin{bmatrix} \Phi\Sigma_U\Phi^T & \Phi\Sigma_\Theta\Phi^T \\ \Phi\Sigma_{U_m}\Phi^T & \Phi\Sigma_{\Theta_m}\Phi^T \end{bmatrix}$$

$$= \begin{bmatrix} \Phi & 0 \\ 0 & \Phi \end{bmatrix} \begin{bmatrix} \Sigma_U & \Sigma_\Theta \\ \Sigma_{U_m} & \Sigma_{\Theta_m} \end{bmatrix} \begin{bmatrix} \Phi & 0 \\ 0 & \Phi \end{bmatrix}^T$$

# Relationships

$$[U] \xrightarrow{SVD} \Sigma_U \rightarrow \lambda_\ell^{[U]}$$

$$[\Theta] \xrightarrow{SVD} \Sigma_\Theta \rightarrow \lambda_\ell^{[\Theta]}$$

$$[U_m] \xrightarrow{SVD} \Sigma_{U_m} \rightarrow \nu_\ell^{[U]}$$

$$[\Theta_m] \xrightarrow{SVD} \Sigma_{\Theta_m} \rightarrow \nu_\ell^{[\Theta]}$$

# Eigenequation for simply-supported boundary

$$\det[SM^S]$$

$$= \prod_{\ell=-(N+1)}^{N} (\lambda_\ell^{[U]} v_\ell^{[\Theta]} - \lambda_\ell^{[\Theta]} v_\ell^{[U]})$$

$$= \prod_{\ell=-(N+1)}^{N} \frac{(-1)^\ell N^2}{16\lambda^2\rho} [J_\ell(\lambda\rho) I_{\ell+1}(\lambda\rho) + I_\ell(\lambda\rho) J_{\ell+1}(\lambda\rho)]$$

$$\{(-1+\nu)(J_{\ell+1}(\lambda\rho) I_\ell(\lambda\rho) + J_\ell(\lambda\rho) I_{\ell+1}(\lambda\rho))$$

$$+ 2\lambda\rho J_\ell(\lambda\rho) I_\ell(\lambda\rho)\} = 0$$

$$\ell = 0, \pm 1, \pm 2, \cdots, \pm(N-1), N$$

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# Determinant (for simply-supported)

$$[SM^F] = \begin{bmatrix} \Phi\Sigma_{U_m}\Phi^T & \Phi\Sigma_{\Theta_m}\Phi^T \\ \Phi\Sigma_{U_v}\Phi^T & \Phi\Sigma_{\Theta_v}\Phi^T \end{bmatrix}$$

$$= \begin{bmatrix} \Phi & 0 \\ 0 & \Phi \end{bmatrix} \begin{bmatrix} \Sigma_{U_m} & \Sigma_{U_m} \\ \Sigma_{U_v} & \Sigma_{\Theta_v} \end{bmatrix} \begin{bmatrix} \Phi & 0 \\ 0 & \Phi \end{bmatrix}^T$$

# Relationships

$$[U_m] \xrightarrow{SVD} \Sigma_{U_m} \rightarrow \nu_\ell^{[U]}$$

$$[\Theta_m] \xrightarrow{SVD} \Sigma_{\Theta_m} \rightarrow \nu_\ell^{[\Theta]}$$

$$[U_v] \xrightarrow{SVD} \Sigma_{U_v} \rightarrow \delta_\ell^{[U]}$$

$$[\Theta_v] \xrightarrow{SVD} \Sigma_{\Theta_v} \rightarrow \delta_\ell^{[\Theta]}$$

# Eigenequation for free boundary

$$\det[SM^F]$$

$$= \prod_{\ell=-(N+1)}^{N} (v_\ell^{[U]}\delta_\ell^{[\Theta]} - v_\ell^{[\Theta]}\delta_\ell^{[U]})$$

$$= \prod_{\ell=-(N+1)}^{N} \frac{(-1)^\ell N^2}{16\lambda^2\rho^4}[J_{\ell+1}(\lambda\rho)I_\ell(\lambda\rho)+J_\ell(\lambda\rho)I_{\ell+1}(\lambda\rho)]$$

$$\{\ell^2(\ell^2-1)(-1+\nu)^2+\lambda^4\rho^4)(J_{\ell+1}(\lambda\rho)I_\ell(\lambda\rho)+J_\ell(\lambda\rho)I_{\ell+1}(\lambda\rho))$$

$$+2\ell\lambda^2\rho^2(1-\ell)(-1+\nu)(J_{\ell+1}(\lambda\rho)I_\ell(\lambda\rho)-J_\ell(\lambda\rho)I_{\ell+1}(\lambda\rho))$$

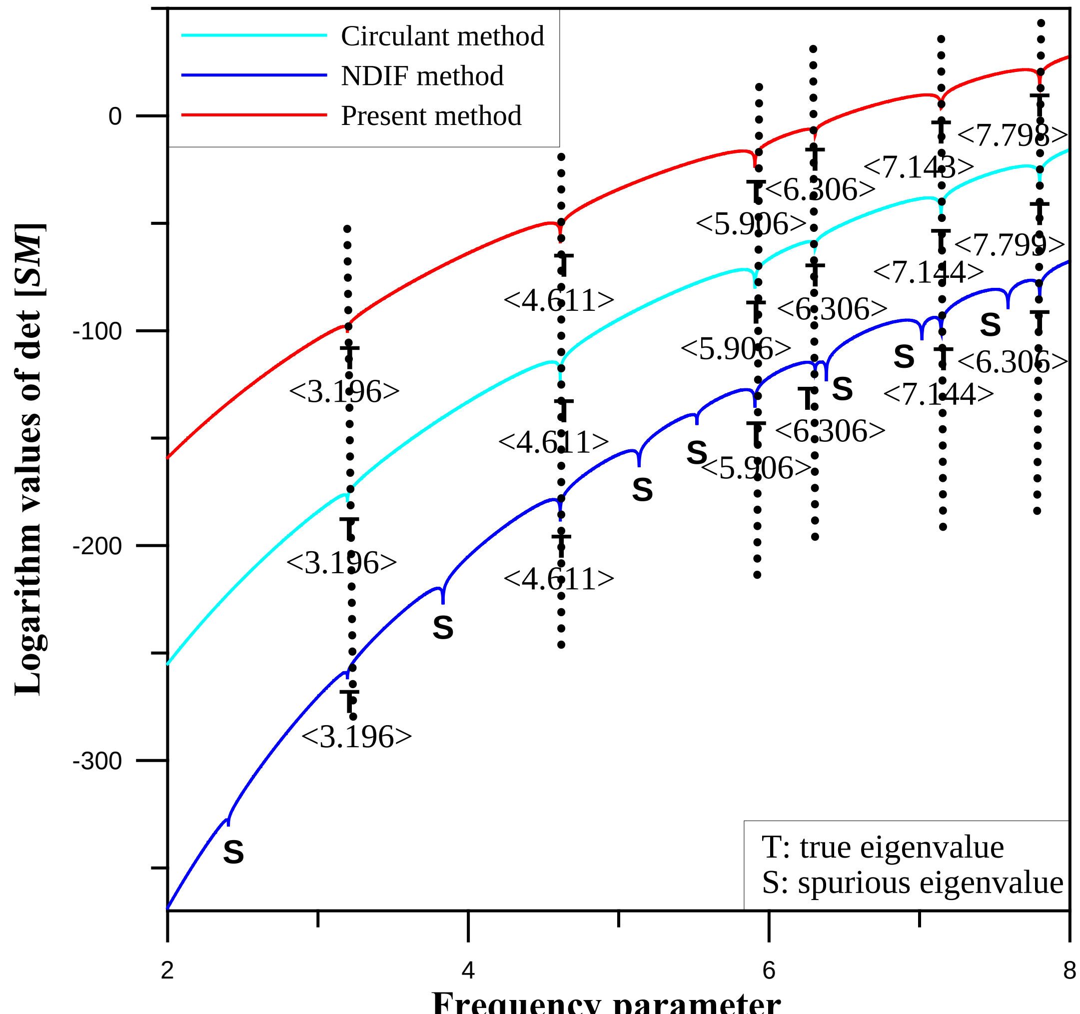
$$+\lambda\rho(-1+\nu)(2\lambda^2\rho^2J_{\ell+1}(\lambda\rho)I_{\ell+1}(\lambda\rho)+4\ell^2(-1+\ell)J_\ell(\lambda\rho)I_\ell(\lambda\rho)=0,$$

$$\ell=0,\pm1,\pm2,\cdots,\pm(N-1),N$$

# Comparisons of the NDIF and present method

| | Kang | Present method |
|---|---|---|
| **Base** | $U(s,x)=J_0(\lambda r)$ <br> $\Theta(s,x)=I_0(\lambda r)$ | $U(s,x)=\frac{1}{8\lambda^2}(J_0(\lambda r)+I_0(\lambda r))$ <br> $\Theta(s,x)=\frac{\partial U(s,x)}{\partial n_s}$ |
| **Clamped plate** | $J_\ell(\lambda r)I_{\ell+1}(\lambda r)+J_{\ell+1}(\lambda r)I_\ell(\lambda r)=0$ | $J_\ell(\lambda r)I_{\ell+1}(\lambda r)+J_{\ell+1}(\lambda r)I_\ell(\lambda r)=0$ |
| | $J_\ell(\lambda r)=0$ | $J_\ell(\lambda r)I_{\ell+1}(\lambda r)+J_{\ell+1}(\lambda r)I_\ell(\lambda r)=0$ |
| **Simply-supported plate** | $\frac{I_{\ell+1}(\lambda r)}{I_\ell(\lambda r)}+\frac{J_{\ell+1}(\lambda r)}{J_\ell(\lambda r)}=\frac{2\lambda r}{(1-\nu)}$ | $\frac{I_{\ell+1}(\lambda r)}{I_\ell(\lambda r)}+\frac{J_{\ell+1}(\lambda r)}{J_\ell(\lambda r)}=\frac{2\lambda r}{(1-\nu)}$ |
| | $J_\ell(\lambda r)=0$ | $J_\ell(\lambda r)I_{\ell+1}(\lambda r)+J_{\ell+1}(\lambda r)I_\ell(\lambda r)=0$ |
| **Treatment** | **Net approach** | **Dual formulation with SVD updating** |

# Circular clamped plate using different methods



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# Comparisons of Leissa and present method

| | Leissa (Kitahara) | Present method |
|---|---|---|
| **Clamped plate** | $J_\ell(\lambda r)I_{\ell+1}(\lambda r)+J_{\ell+1}(\lambda r)I_\ell(\lambda r)=0$ | $J_\ell(\lambda r)I_{\ell+1}(\lambda r)+J_{\ell+1}(\lambda r)I_\ell(\lambda r)=0$ |
| **Simply-supported plate** | $\frac{I_{\ell+1}(\lambda r)}{I_\ell(\lambda r)}+\frac{J_{\ell+1}(\lambda r)}{J_\ell(\lambda r)}=\frac{2\lambda r}{(1-\nu)}$ | $\frac{I_{\ell+1}(\lambda r)}{I_\ell(\lambda r)}+\frac{J_{\ell+1}(\lambda r)}{J_\ell(\lambda r)}=\frac{2\lambda r}{(1-\nu)}$ |
| **Free plate** J | $\frac{\lambda^2 J_\ell(\lambda\rho)+(1-\nu)[\lambda J'_\ell(\lambda\rho)-\ell^2 J_\ell(\lambda\rho)]}{\lambda^2 I_\ell(\lambda\rho)-(1-\nu)[\lambda I'_\ell(\lambda\rho)-\ell^2 I_\ell(\lambda\rho)]}$ $=\frac{\lambda^3 I'_\ell(\lambda\rho)+(1-\nu)\ell^2[\lambda J'_\ell(\lambda\rho)-J_\ell(\lambda\rho)]}{\lambda^3 I'_\ell(\lambda\rho)-(1-\nu)\ell^2[\lambda I'_\ell(\lambda\rho)-I_\ell(\lambda\rho)]}$ | $\{\ell^2(\ell^2-1)(-1+\nu)^2+\lambda^4\rho^4)(J_{\ell+1}(\lambda\rho)I_\ell(\lambda\rho)$ $+J_\ell(\lambda\rho)I_{\ell+1}(\lambda\rho))+2\ell\lambda^2\rho^2(1-\ell)(-1+\nu)$ $(J_{\ell+1}(\lambda\rho)I_\ell(\lambda\rho)-J_\ell(\lambda\rho)I_{\ell+1}(\lambda\rho))+\lambda\rho(-1+\nu)$ $(2\lambda^2\rho^2 J_{\ell+1}(\lambda\rho)I_{\ell+1}(\lambda\rho)+4\ell^2(-1+\ell)J_\ell(\lambda\rho)I_\ell(\lambda\rho)$ $=0$ |

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4. SVD updating terms
5. Conclusions

# SVD updating terms (clamped case)

$$\left[SM^{C}\right]\begin{Bmatrix}\phi\\ \psi\end{Bmatrix}=\begin{bmatrix}U & \Theta\\ U_{\theta} & \Theta_{\theta}\end{bmatrix}\begin{Bmatrix}\phi\\ \psi\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix}$$

$$\left[SM_{1}^{C}\right]\begin{Bmatrix}\phi'\\ \psi'\end{Bmatrix}=\begin{bmatrix}M & V\\ M_{\theta} & V_{\theta}\end{bmatrix}\begin{Bmatrix}\phi'\\ \psi'\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix}$$

# SVD updating terms (clamped case)

$$[C]=\begin{bmatrix}(SM^{C})^{T}\\(SM_{1}^{C})^{T}\end{bmatrix}$$

$$=\begin{bmatrix}\Phi & 0 & 0 & 0\\0 & \Phi & 0 & 0\\0 & 0 & \Phi & 0\\0 & 0 & 0 & \Phi\end{bmatrix}\begin{bmatrix}\Sigma_{U} & \Sigma_{U_\theta}\\\Sigma_{\Theta} & \Sigma_{\Theta_\theta}\\\Sigma_{M} & \Sigma_{M_\theta}\\\Sigma_{V} & \Sigma_{V_\theta}\end{bmatrix}_{8N\times 4N}\begin{bmatrix}\Phi^{-1} & 0\\0 & \Phi^{-1}\end{bmatrix}$$

# SVD updating terms (clamped case)

**Based on the least squares**

$$[C]^T[C]=\begin{bmatrix}\Phi & 0\\ 0 & \Phi\end{bmatrix}[D]_{4N\times 4N}\begin{bmatrix}\Phi^{-1} & 0\\ 0 & \Phi^{-1}\end{bmatrix}$$

$$\det\left[[C]^T[C]\right]=\det[D]$$

$$=\prod_{\ell=-(N-1)}^{N}[(\lambda_\ell^{[U]}\mu_\ell^{[\Theta]}-\mu_\ell^{[U]}\lambda_\ell^{[\Theta]})^2+(\lambda_\ell^{[U]}\mu_\ell^{[M]}-\mu_\ell^{[U]}\lambda_\ell^{[M]})^2$$

$$+(\lambda_\ell^{[U]}\mu_\ell^{[V]}-\mu_\ell^{[U]}\lambda_\ell^{[V]})^2+(\lambda_\ell^{[\Theta]}\mu_\ell^{[M]}-\mu_\ell^{[\Theta]}\lambda_\ell^{[M]})^2$$

$$+(\lambda_\ell^{[\Theta]}\mu_\ell^{[V]}-\mu_\ell^{[\Theta]}\lambda_\ell^{[V]})^2+(\lambda_\ell^{[M]}\mu_\ell^{[V]}-\mu_\ell^{[M]}\lambda_\ell^{[V]})^2]$$

# SVD updating terms (clamped case)

The only possibility for zero determinant of [*D*]

$$(\lambda_\ell^{[U]}\mu_\ell^{[\Theta]}-\mu_\ell^{[U]}\lambda_\ell^{[\Theta]})=0,\quad (\lambda_\ell^{[U]}\mu_\ell^{[M]}-\mu_\ell^{[U]}\lambda_\ell^{[M]})=0,$$

$$(\lambda_\ell^{[U]}\mu_\ell^{[V]}-\mu_\ell^{[U]}\lambda_\ell^{[V]})=0,\quad (\lambda_\ell^{[\Theta]}\mu_\ell^{[M]}-\mu_\ell^{[\Theta]}\lambda_\ell^{[M]})=0,$$

$$(\lambda_\ell^{[\Theta]}\mu_\ell^{[V]}-\mu_\ell^{[\Theta]}\lambda_\ell^{[V]})=0,\quad (\lambda_\ell^{[M]}\mu_\ell^{[V]}-\mu_\ell^{[M]}\lambda_\ell^{[V]})=0.$$

at the same time for the same $\ell$ .

The common term is

$$J_\ell(\lambda r)I_{\ell+1}(\lambda r)+J_{\ell+1}(\lambda r)I_\ell(\lambda r)=0$$

True eigenequation

# Outlines

1. Introduction
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# Conclusions

1. Since any two combinations of the four types of potentials, six options( $C_2^4$ ) were considered.

2. Spurious eigenequation only depends on the adopted kernel function, while the true eigenequation is relevant to the specified boundary condition.

3. True eigenequation can be extract out by using SVD updating term.

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# Thanks for your kind attention