(a) By choosing T-complete set：$\{1, rcos(\theta), r^2cos(2\theta), rsin(\theta), r^2sin(2\theta)\}$

$$\theta_i = \left\{0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}\right\}$$

$$u(r,\theta) = \sum_{n=0}^{2} A_n\ r^n cos(n\theta) + \sum_{n=1}^{2} B_n\ r^n sin(n\theta) \qquad \text{then build } [K]\{a\} = \{b\}$$

$$[K] = \begin{bmatrix} 1. & 1. & 1. & 0. & 0. \\ 1. & 0.309017 & -0.809017 & 0.951057 & 0.587785 \\ 1. & -0.809017 & 0.309017 & 0.587785 & -0.951057 \\ 1. & -0.809017 & 0.309017 & -0.587785 & 0.951057 \\ 1. & 0.309017 & -0.809017 & -0.951057 & -0.587785 \end{bmatrix} \qquad \{b\} = \begin{bmatrix} 1. \\ -0.809017 \\ 0.309017 \\ 0.309017 \\ -0.809017 \end{bmatrix}$$

$$\{a\} = \{A_0, A_1, A_2, B_1, B_2\}^T$$

Finally, we get $\{A_0, A_1, A_2, B_1, B_2\}^T = \{0, 0, 1, 0, 0\}^T$

(b) By choosing T-complete set：

$$\{1, rcos(\theta), r^2cos(2\theta), r^3cos(3\theta), r^4cos(4\theta), rsin(\theta), r^2sin(2\theta), r^3sin(3\theta), r^4sin(4\theta)\}$$

$$\theta_i = \left\{0, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{6\pi}{9}, \frac{8\pi}{9}, \frac{10\pi}{9}, \frac{12\pi}{9}, \frac{14\pi}{9}, \frac{16\pi}{9}\right\}$$

$$u(r,\theta) = \sum_{n=0}^{4} A_n\ r^n cos(n\theta) + \sum_{n=1}^{4} B_n\ r^n sin(n\theta) \qquad \text{then build } [K]\{a\} = \{b\}$$

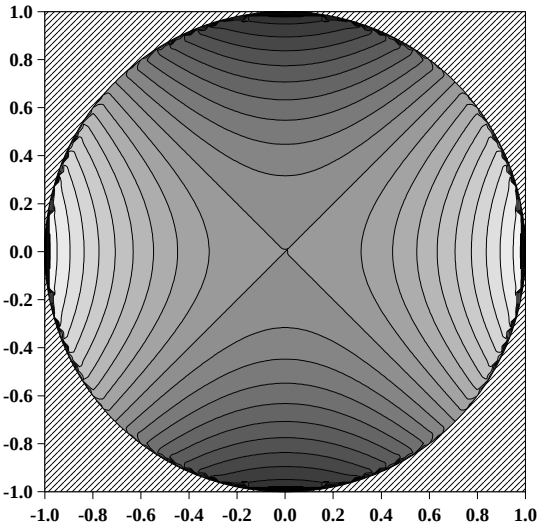
$$[K] = \begin{bmatrix} 1. & 1. & 1. & 1. & 1. & 0. & 0. & 0. & 0. \\ 1. & 0.766044 & 0.173648 & -0.5 & -0.939693 & 0.642788 & 0.984808 & 0.866025 & 0.34202 \\ 1. & 0.173648 & -0.939693 & -0.5 & 0.766044 & 0.984808 & 0.34202 & -0.866025 & -0.642788 \\ 1. & -0.5 & -0.5 & 1. & -0.5 & 0.866025 & -0.866025 & 0. & 0.866025 \\ 1. & -0.939693 & 0.766044 & -0.5 & 0.173648 & 0.34202 & -0.642788 & 0.866025 & -0.984808 \\ 1. & -0.939693 & 0.766044 & -0.5 & 0.173648 & -0.34202 & 0.642788 & -0.866025 & 0.984808 \\ 1. & -0.5 & -0.5 & 1. & -0.5 & -0.866025 & 0.866025 & 0. & -0.866025 \\ 1. & 0.173648 & -0.939693 & -0.5 & 0.766044 & -0.984808 & -0.34202 & 0.866025 & 0.642788 \\ 1. & 0.766044 & 0.173648 & -0.5 & -0.939693 & -0.642788 & -0.984808 & -0.866025 & -0.34202 \end{bmatrix}$$

$$\{a\} = \{A_0, A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4\}^T \qquad \{b\} = \begin{bmatrix} 1. \\ -0.5 \\ -0.5 \\ 1. \\ -0.5 \\ -0.5 \\ 1. \\ -0.5 \\ -0.5 \end{bmatrix}$$

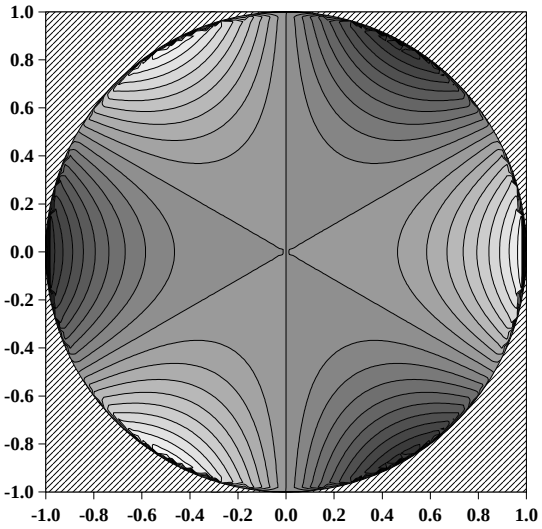
Finally, we get $\{A_0, A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4\}^T = \{0,0,0,1,0,0,0,0,0\}^T$

The exact solution $u(r,\theta) = r^3 cos(3\theta),\ \mathrm{r} \le 1$

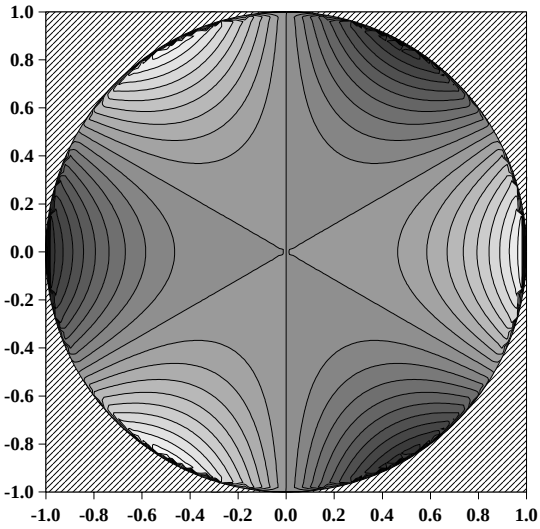
Plot the field distribution for Trefftz method and exact solution.



Trefftz method (5 sets)



Trefftz method (9 sets)



Exact solution