班級：結構組碩一A 學號：M93520008 姓名：吳安傑

國立台灣海洋大學河海工程研究所BEM作業四（2004）

$$\int_a^b \frac{2 x_2^2}{((x_1 - s_1)^2 + x_2^2)^2}\, ds_1$$

$$= \frac{\text{ArcTan}\left[\frac{a-x_1}{x_2}\right]}{x_2} + \frac{(a - x_1)}{(a - x_1)^2 + x_2^2} - \frac{\text{ArcTan}\left[\frac{b-x_1}{x_2}\right]}{x_2} - \frac{(b - x_1)}{(b - x_1)^2 + x_2^2}$$

$$= \frac{\frac{\pi}{2} - \text{ArcTan}\left[\frac{x_2}{a-x_1}\right]}{x_2} + \frac{(a - x_1)}{(a - x_1)^2 + x_2^2} - \frac{\frac{\pi}{2} - \text{ArcTan}\left[\frac{x_2}{b-x_1}\right]}{x_2} - \frac{(b - x_1)}{(b - x_1)^2 + x_2^2}$$

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Simplify[D[π/2 - ArcTan[x2/(a - x1)], x2]]
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$$\frac{-a + x_1}{a^2 - 2 a x_1 + x_1^2 + x_2^2}$$

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Simplify[D[π/2 - ArcTan[x2/(b - x1)], x2]]
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$$\frac{-b + x_1}{b^2 - 2 b x_1 + x_1^2 + x_2^2}$$

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Limit[(-a + x1)/(a^2 - 2 a x1 + x1^2 + x2^2) + (a - x1)/((a - x1)^2 + x2^2) - (-b + x1)/(b^2 - 2 b x1 + x1^2 + x2^2) - (b - x1)/((b - x1)^2 + x2^2), x2 → 0]
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0

$$\to \lim_{x_2 \to 0} \left(\int_a^b \frac{2 x_2^2}{((x_1 - s_1)^2 + x_2^2)^2}\, ds_1 \right)$$

$$= \lim_{x_2 \to 0} \left(\frac{-a + x_1}{a^2 - 2 a x_1 + x_1^2 + x_2^2} + \frac{(a - x_1)}{(a - x_1)^2 + x_2^2} - \frac{-b + x_1}{b^2 - 2 b x_1 + x_1^2 + x_2^2} - \frac{(b - x_1)}{(b - x_1)^2 + x_2^2} \right)$$

$$= 0$$