班級：結構組碩一A 學號：M93520008 姓名：吳安傑
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Fouries Transform $\rightarrow u(k) = \int_{-\infty}^{\infty} U(x)\, e^{-ikx}\, dx$

$\rightarrow (ik)^4 u(k) = \int_{-\infty}^{\infty} \frac{d^4 U(x)}{dx^4}\, e^{-ikx}\, dx$

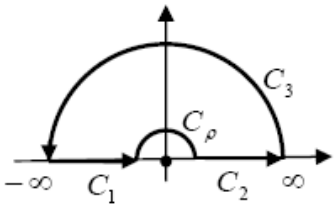
$\rightarrow k^4 u(k) = \int_{-\infty}^{\infty} \delta(x-s)\, e^{-ikx}\, dx = e^{-iks}$

$\rightarrow u(k) = \frac{1}{k^4}\, e^{-iks}$

Inverse Fouries Transform $\rightarrow U(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} u(x)\, e^{ikx}\, dk$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{k^4}\, e^{-iks}\, e^{ikx}\, dk$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{k^4}\, e^{ik(x-s)}\, dk \rightarrow \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{z^4}\, e^{iz(x-s)}\, dz$$



$x > s$

$$\oint \frac{1}{z^4}\, e^{iz(x-s)}\, dz = \int_{C_1+C_\rho+C_2+C_3} dz = \int_{C_1+C_2} dz + \int_{C_\rho} dz + \underbrace{\int_{C_3} dz}_{=0} = 0$$

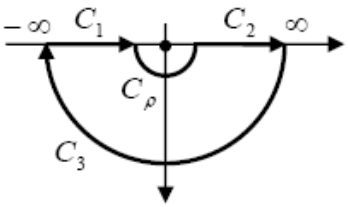
$$\int_{C_\rho} \frac{1}{z^4}\, e^{iz(x-s)}\, dz = \int_{C_\rho} \frac{1 + i(-s+x)z - \frac{1}{2}(-s+x)^2 z^2 + -\frac{1}{6} i (-s+x)^3 z^3 + \ldots.}{z^4}\, dz$$

$$= -\frac{2}{3\rho^3} + \frac{(x-s)^2}{\rho} - \frac{\pi (x-s)^3}{6}$$

$$\rightarrow \int_{C_1+C_2} dz + \int_{C_\rho} dz + \underbrace{\int_{C_3} dz}_{=0} = \lim_{\rho \to 0} \int_{-\infty}^{-\rho} + \int_{\rho}^{\infty} dz - \frac{2}{3\rho^3} + \frac{(x-s)^2}{\rho} - \frac{\pi (x-s)^3}{6} = 0$$

$$\rightarrow \lim_{\rho \to 0} \int_{-\infty}^{-\rho} + \int_{\rho}^{\infty} dz - \frac{2}{3\rho^3} + \frac{(x-s)^2}{\rho} = \frac{\pi (x-s)^3}{6}$$

$$\rightarrow U(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{z^4}e^{iz(x-s)}\,dz = \frac{1}{2\pi}\left(\frac{\pi(x-s)^3}{6}\right) = \frac{(x-s)^3}{12}$$



$x < s$

$$\oint\frac{1}{z^4}e^{iz(x-s)}\,dz = \int_{C_1+C_\rho+C_2+C_3}dz = \int_{C_1+C_2}dz + \int_{C_\rho}dz + \underbrace{\int_{C_3}dz}_{=0} = 0$$

$$\rightarrow \int_{C_1+C_2}dz + \int_{C_\rho}dz + \underbrace{\int_{C_3}dz}_{=0} = \lim_{\rho\to 0}\int_{-\infty}^{-\rho} + \int_{\rho}^{\infty}dz - \frac{2}{3\rho^3} + \frac{(x-s)^2}{\rho} + \frac{\pi(x-s)^3}{6} = 0$$

$$\rightarrow \lim_{\rho\to 0}\int_{-\infty}^{-\rho} + \int_{\rho}^{\infty}dz - \frac{2}{3\rho^3} + \frac{(x-s)^2}{\rho} = -\frac{\pi(x-s)^3}{6}$$

$$\rightarrow U(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{z^4}e^{iz(x-s)}\,dz = \frac{1}{2\pi}\left(-\frac{\pi(x-s)^3}{6}\right) = -\frac{(x-s)^3}{12}$$