班級：______________學號：____M93520010____姓名：_____陳柏源_____

國立台灣海洋大學河海工程研究所 BEM2004 第 6 次作業

In the course, we have derived the constraint among u(0), u(1), t(0) and t(1) by using the null field integral equation ($s = 0^-$ and 1^+) for UT and LM equations.

Please derive the constraint by using the boundary integral equations for UT and LM ($s = 0^+$ and 1^-).

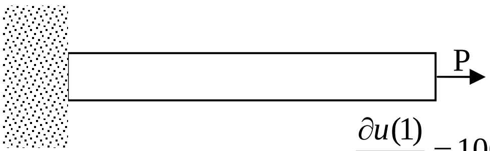
$$\frac{d^2u(x)}{dx^2} = 0$$

$$\frac{d^2v(x)}{dx^2} = \delta(x-s)$$

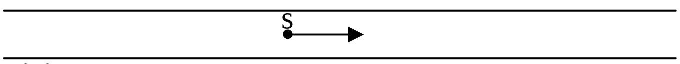
$$v(x) = U(x,s) = \frac{1}{2}|x-s|$$

欲解問題

$u(x) = 0$



$u(0) = 0$ $\qquad \frac{\partial u(1)}{\partial x} = 100$



基本解

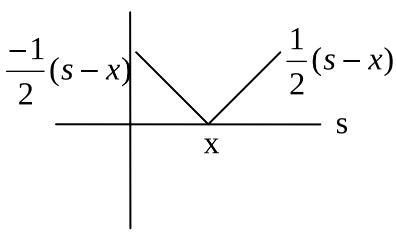
$v(x) = 0$

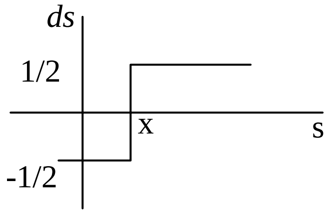
符號交換

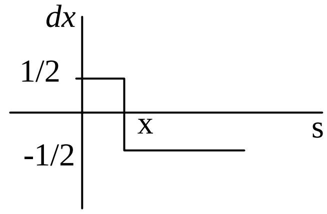
$$v(s) = U(s,x) = \frac{1}{2}|s-x|$$

$$\frac{dU(s,x)}{ds} = \begin{cases} \frac{1}{2}, s > x \\ -\frac{1}{2}, s < x \end{cases}$$

$$\frac{dU(s,x)}{dx} = \begin{cases} -\frac{1}{2}, s > x \\ \frac{1}{2}, s < x \end{cases}$$







BEM H.W .006 M93520010 陳柏源

ANS.

$$\int_0^1 u(x)\,\frac{d^2 v(x)}{dx^2}\,dx = \int_0^1 u(x)\,\delta(x-s)\,dx$$

左式 $= u(x)\,\frac{dv(x)}{dx}\Big|_0^1 - \int_0^1 \frac{du(x)}{dx}\,\frac{dv(x)}{dx}\,dx$

$\quad = u(x)\,\frac{dv(x)}{dx}\Big|_0^1 - \left[\frac{du(x)}{dx}\,v(x)\Big|_0^1 - \int_0^1 \frac{d^2u(x)}{dx^2}\,v(x)\,dx\right]$

$\quad = u(x)\,\frac{dv(x)}{dx}\Big|_0^1 - \frac{du(x)}{dx}\,v(x)\Big|_0^1$

右式 $= \int_0^1 u(x)\,\delta(x-s)\,dx$

$\quad = u(s)$

$\Rightarrow u(s) = u(x)\,\frac{dU(x,s)}{dx}\Big|_0^1 - \frac{du(x)}{dx}\,U(x,s)\Big|_0^1$

$\Rightarrow$ 符號互換

$u(x) = u(s)\,\frac{dU(s,x)}{ds}\Big|_0^1 - \frac{du(s)}{ds}\,U(s,x)\Big|_0^1$

域內點邊界積分方程 $\Rightarrow u(x) = u(s)\,\frac{dU(s,x)}{ds}\Big|_0^1 - \frac{du(s)}{ds}\,U(s,x)\Big|_0^1$

域外點邊界積分方程 $\Rightarrow 0 = u(s)\,\frac{dU(s,x)}{ds}\Big|_0^1 - \frac{du(s)}{ds}\,U(s,x)\Big|_0^1$

(A)

域內點邊界積分方程 (U.T)

$$\begin{cases} u(0^+) = u(s)\,\frac{dU(s,0^+)}{ds}\Big|_0^1 - \frac{du(s)}{ds}\,U(s,0^+)\Big|_0^1 \;\ldots\ldots\;(1) \\ u(1^-) = u(s)\,\frac{dU(s,1^-)}{ds}\Big|_0^1 - \frac{du(s)}{ds}\,U(s,1^-)\Big|_0^1 \;\ldots\ldots\;(2) \end{cases}$$

$$\begin{cases} \frac{dU(1,1^-)}{ds} = \frac{1}{2} \\ \frac{dU(1,0^+)}{ds} = \frac{1}{2} \\ \frac{dU(0,1^-)}{ds} = \frac{-1}{2} \\ \frac{dU(0,0^+)}{ds} = \frac{-1}{2} \end{cases} \qquad \begin{cases} U(1,1^-) = 0 \\ U(1,0^+) = \frac{1}{2} \\ U(0,1^-) = \frac{1}{2} \\ U(0,0^+) = 0 \end{cases}$$

(1) 式得

$u(0^+) = u(s)\,\frac{dU(s,0^+)}{ds}\Big|_0^1 - \frac{du(s)}{ds}\,U(s,0^+)\Big|_0^1$

$\Rightarrow u(0) = u(1)\,\frac{dU(1,0^+)}{ds} - t(1)\,U(1,0^+) - u(0)\,\frac{dU(0,0^+)}{ds} + t(0)\,U(0,0^+)$

$\Rightarrow u(0) = u(1) * \frac{1}{2} - 100 * \frac{1}{2}$

$\Rightarrow 0 = u(1) * \frac{1}{2} - 100 * \frac{1}{2}$

$\Rightarrow u(1) = 100$

(2) 式得

$u(1^-) = u(s)\,\frac{dU(s,1^-)}{ds}\Big|_0^1 - \frac{du(s)}{ds}\,U(s,1^-)\Big|_0^1$

$$\Rightarrow u(1) = u(1)\frac{dU(1,1^-)}{ds} - t(1)\,U(1,1^-) - u(0)\frac{dU(0,1^-)}{ds} + t(0)\,U(0,1^-)$$

$$\Rightarrow u(1) = u(1) * \frac{1}{2} + t(0) * \frac{1}{2}$$

$$\Rightarrow t(0) = 100$$

$$\begin{cases} U(1,x) = \frac{1}{2}(1-x) \\ U(0,x) = \frac{1}{2}(x) \end{cases}, \begin{cases} \frac{dU(1,x)}{ds} = \frac{1}{2} \\ \frac{dU(0,x)}{ds} = -\frac{1}{2} \end{cases}$$

$$u(x) = u(1)\frac{dU(1,x)}{ds} - t(1)\,U(1,x) - u(0)\frac{dU(0,x)}{ds} + t(0)\,U(0,x)$$

$$= 100 * \frac{1}{2} - 100 * \frac{1}{2}(1-x) + 100 * \frac{1}{2}x$$

$$= 50 - 50 + 50x + 50x$$

$$= 100x$$

(B)

域外點 (L.M)

$$\frac{du(x)}{dx} = u(s)\,M(s,x)\Big|_0^1 - t(s)\,L(s,x)\Big|_0^1$$

$$\begin{cases} t(0^-) = u(s)\,M(s,0^-)\Big|_0^1 - t(s)\,L(s,0^-)\Big|_0^1 \;\ldots\ldots(3) \\ t(1^+) = u(s)\,M(s,1^+)\Big|_0^1 - t(s)\,L(s,1^+)\Big|_0^1 \;\ldots\ldots(4) \end{cases}$$

$$\begin{cases} M(1,0^-) = 0 \\ M(0,0^-) = 0 \\ M(1,1^+) = 0 \\ M(0,1^+) = 0 \end{cases} \qquad \begin{cases} L(1,0^-) = \frac{-1}{2} \\ L(0,0^-) = \frac{-1}{2} \\ L(1,1^+) = \frac{1}{2} \\ L(0,1^+) = \frac{1}{2} \end{cases}$$

(3) 式得

$$t(0^-) = -t(1)\,L(1,0^-) + t(0)\,L(0,0^-)$$

$$0 = -100 * \frac{-1}{2} + t(0) * \frac{-1}{2}$$

$$t(0) = 100. \ldots\ldots(5)$$

(4) 式得

$$t(1^+) = u(1)\,M(1,1^+) - t(1)\,L(1,1^+) - u(0)\,M(0,1^+) + t(0)\,L(0,1^+)$$

$$0 = -100 * \frac{1}{2} + t(0) * \frac{1}{2}$$

$$t(0) = 100. \ldots\ldots(6)$$

(5).(6) 相依.無法求解