班級：____________學號：_____________姓名：_____________

國立台灣海洋大學河海工程研究所 BEM2004 第 9 次作業

1. In the course, we used null-field integral equation

$$0=\int_B T^E(s,x)u(s)dB(s)-\int_B U^E(s,x)t(s)dB(s) \quad \text{to obtain} \quad t(1,\theta)=\cos\theta .$$

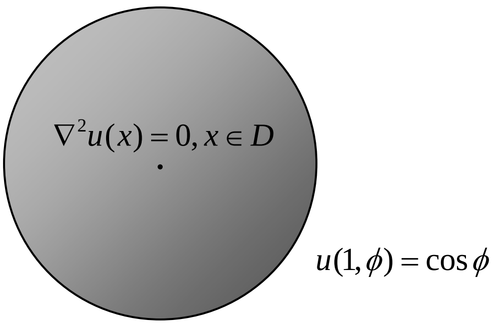
Substituting the degenerate kernels $U^I(s,x)$, $T^I(s,x)$, $u(1,\theta)=\cos\theta$, and

$t(1,\theta)=\cos\theta$ to $2\pi u(x)=\int_B T^I(s,x)u(s)dB(s)-\int_B U^I(s,x)t(s)dB(s)$ to obtain the

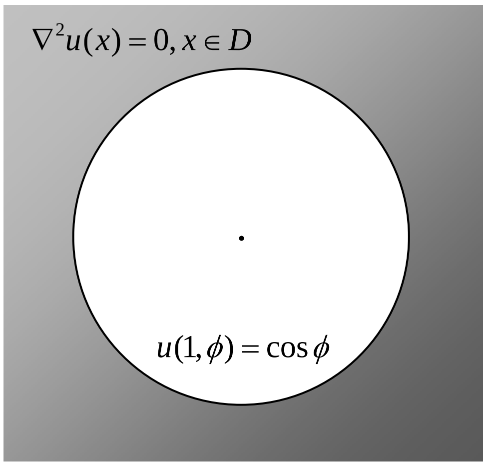
exact solution $u(\rho,\phi)$, $0\le\rho\le 1$, $0<\phi<2\pi$, where

$$U(s,x)=\ln r=\begin{cases} U^I=\ln R-\sum_{m=1}^{\infty}\frac{1}{m}(\frac{\rho}{R})^m\left(\cos m(\theta-\phi)\right), & R>\rho \\ U^E=\ln\rho-\sum_{m=1}^{\infty}\frac{1}{m}(\frac{R}{\rho})^m\left(\cos m(\theta-\phi)\right), & R<\rho \end{cases}$$

Repeat the procedure by approaching the boundary from the domain for both the interior and exterior problems.



Exact solution
$u(\rho,\phi)=\rho\cos\phi, 0<\rho<1, 0<\phi<2\pi$



Exact solution
$u(\rho,\phi)=\frac{1}{\rho}\cos\phi, \rho>1, 0<\phi<2\pi$