班級：結構組碩一A 學號：M93520008 姓名：吳安傑
國立台灣海洋大學河海工程研究所BEM作業九（2004） Filename：BEM09s.nb

$x \in D$（域內點邊界積分方程式）

$$2\pi u(x) = \int_B \frac{\partial U(s,x)}{\partial n_s} u(s)\, dB(s) - \int_B U(s,x) \frac{\partial u(s)}{\partial n_s}\, dB(s)$$

$$\rightarrow 2\pi u(x) = \int_B T(s,x)\, u(s)\, dB(s) - \int_B U(s,x)\, t(s)\, dB(s)$$

$X \notin D$（域外點邊界積分方程式）

$$0 = \int_B \frac{\partial U(s,x)}{\partial n_s} u(s)\, dB(s) - \int_B U(s,x) \frac{\partial u(s)}{\partial n_s}\, dB(s)$$

$$\rightarrow 0 = \int_B T(s,x)\, u(s)\, dB(s) - \int_B U(s,x)\, t(s)\, dB(s)$$

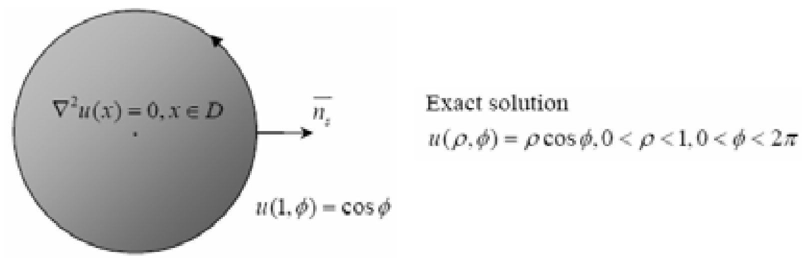
$x \in B$

$$\pi u(x) = \text{C.P.V} \int T(s,x)\, u(s)\, dB(s) - \int U(s,x)\, t(s)\, dB(s)$$

$$U(s,x) = \ln r = \begin{cases} U^I = \ln R - \sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{\rho}{R}\right)^m \cos m(\theta-\phi), & R > \rho \\ U^E = \ln \rho - \sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{R}{\rho}\right)^m \cos m(\theta-\phi), & R < \rho \end{cases}$$

$$T(s,x) = \frac{\partial \ln r}{\partial n_s} = \frac{\partial \ln r}{\partial R} = \begin{cases} T^I = \frac{1}{R} + \sum_{m=1}^{\infty} \frac{\rho^m}{R^{m+1}} \cos m(\theta-\phi), & R > \rho \\ T^E = -\sum_{m=1}^{\infty} \frac{R^{m-1}}{\rho^m} \cos m(\theta-\phi), & R < \rho \end{cases}$$

▪**Interior problems**



$$2\pi u(x) = \int_B \frac{\partial U(s,x)}{\partial n_s} u(s)\, dB(s) - \int_B U(s,x) \frac{\partial u(s)}{\partial n_s}\, dB(s)$$

$$\rightarrow 2\pi u(x) = \int_B T(s,x)\, u(s)\, dB(s) - \int_B U(s,x)\, t(s)\, dB(s), \text{ let } \rho = 1^- \rightarrow R > \rho$$

$$\rightarrow 2\pi u(x) = \int_B T^I(s,x)\, u(s)\, dB(s) - \int_B U^I(s,x)\, t(s)\, dB(s)$$

$$\rightarrow 2\pi u(x) = \int_0^{2\pi} \left\{ \frac{1}{R} + \sum_{m=1}^{\infty} \frac{\rho^m}{R^{m+1}} \cos m(\theta-\phi) \right\} \cos\theta\, d\theta$$
$$- \int_0^{2\pi} \left\{ \ln R - \sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{\rho}{R}\right)^m \cos m(\theta-\phi) \right\} \left(\sum_{m=1}^{\infty} a_m \cos m\theta + b_m \sin m\theta \right) d\theta$$

$$\to 2\pi u(x) = \int_0^{2\pi}\left\{1+\sum_{m=1}^{\infty}\cos m(\theta-\phi)\right\}\cos\theta\, d\theta - \int_0^{2\pi}\left\{0-\sum_{m=1}^{\infty}\frac{1}{m}\cos m(\theta-\phi)\right\}\left(\sum_{m=1}^{\infty} a_m \cos m\theta + b_m \sin m\theta\right) d\theta$$

$$\to 2\pi u(x) = \int_0^{2\pi}\sum_{m=1}^{\infty}\cos m(\theta-\phi)\cos\theta\, d\theta + \int_0^{2\pi}\sum_{m=1}^{\infty}\frac{1}{m}(\cos m\theta\cos m\phi\, a_m \cos m\theta + \sin m\theta\sin m\phi\, b_m \sin m\theta)\, d\theta$$

$$\to 2\pi u(x) = \int_0^{2\pi}\cos(\theta-\phi)\cos\theta\, d\theta + \int_0^{2\pi}(\cos\theta\cos\phi\, a_m \cos\theta + \sin\theta\sin\phi\, b_m \sin\theta)\, d\theta$$

$$\to 2\pi u(x) = \pi\cos\phi + a_1\pi\cos\phi + b_1\pi\sin\phi$$

$u(1,\phi)=\cos\phi$代入 $\to 2\pi\cos\phi = \pi\cos\phi + a_1\pi\cos\phi + b_1\pi\sin\phi$

$\to a_1 = 1,\ b_1 = 0,\ t(1,\theta) = \cos\theta$

$$2\pi u(x) = \int_B T^I(s,x)\, u(s)\, dB(s) - \int_B U^I(s,x)\, t(s)\, dB(s)$$

$$\to 2\pi u(x) = \int_0^{2\pi}\left\{\frac{1}{R}+\sum_{m=1}^{\infty}\frac{\rho^m}{R^{m+1}}\cos m(\theta-\phi)\right\}\cos\theta\, d\theta - \int_0^{2\pi}\left\{\ln R-\sum_{m=1}^{\infty}\frac{1}{m}\left(\frac{\rho}{R}\right)^m\cos m(\theta-\phi)\right\}\cos(\theta)\, d\theta$$

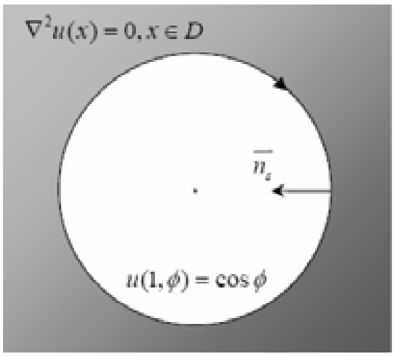
$$\to 2\pi u(x) = \int_0^{2\pi}\left\{1+\sum_{m=1}^{\infty}\rho^m\cos m(\theta-\phi)\right\}\cos\theta\, d\theta - \int_0^{2\pi}\left\{0-\sum_{m=1}^{\infty}\frac{1}{m}\rho^m\cos m(\theta-\phi)\right\}\cos(\theta)\, d\theta$$

$$\to 2\pi u(x) = \int_0^{2\pi}\rho\cos(\theta-\phi)\cos\theta\, d\theta + \int_0^{2\pi}\rho\cos(\theta-\phi)\cos(\theta)\, d\theta$$

$$\to 2\pi u(x) = 2\int_0^{2\pi}\rho\cos(\theta-\phi)\cos\theta\, d\theta$$

$$\to 2\pi u(x) = 2\rho\pi\cos\phi \to u(x) = u(\rho,\phi) = \rho\cos\phi$$

▪**Exterior problems**



Exact solution

$$u(\rho,\phi) = \frac{1}{\rho}\cos\phi,\ \rho>1,\ 0<\phi<2\pi$$

$$2\pi u(x) = \int_B \frac{\partial U(s,x)}{\partial n_s} u(s)\, dB(s) - \int_B U(s,x) \frac{\partial u(s)}{\partial n_s}\, dB(s)$$

$$\to 2\pi u(x) = \int_B T(s,x)\, u(s)\, dB(s) - \int_B U(s,x)\, t(s)\, dB(s),\ \ \text{let } \rho = 1^+ \to R < \rho$$

$$\to 2\pi u(x) = \int_B -T^E(s,x)\, u(s)\, dB(s) - \int_B U^E(s,x)\, t(s)\, dB(s)$$

$$\to 2\pi u(x) = \int_0^{2\pi} -\left\{-\sum_{m=1}^{\infty} \frac{R^{m-1}}{\rho^m} \cos m(\theta-\phi)\right\} \cos\theta\, d\theta$$
$$- \int_0^{2\pi} \left\{\ln\rho - \sum_{m=1}^{\infty} \frac{1}{m}\left(\frac{R}{\rho}\right)^m \cos m(\theta-\phi)\right\} \left(\sum_{m=1}^{\infty} a_m \cos m\theta + b_m \sin m\theta\right) d\theta$$

$$\to 2\pi u(x) = \int_0^{2\pi} \sum_{m=1}^{\infty} \cos m(\theta-\phi) \cos\theta\, d\theta$$
$$- \int_0^{2\pi} \left\{0 - \sum_{m=1}^{\infty} \frac{1}{m} \cos m(\theta-\phi)\right\} \left(\sum_{m=1}^{\infty} a_m \cos m\theta + b_m \sin m\theta\right) d\theta$$

$$\to 2\pi u(x) = \int_0^{2\pi} \sum_{m=1}^{\infty} \cos m(\theta-\phi) \cos\theta\, d\theta$$
$$- \int_0^{2\pi} \sum_{m=1}^{\infty} \frac{1}{m} (\cos m\theta \cos m\phi\, a_m \cos m\theta + \sin m\theta \sin m\phi\, b_m \sin m\theta)\, d\theta$$

$$\to 2\pi u(x) = \int_0^{2\pi} \cos(\theta-\phi) \cos\theta\, d\theta - \int_0^{2\pi} (\cos\theta\cos\phi\, a_m \cos\theta + \sin\theta\sin\phi\, b_m \sin\theta)\, d\theta$$

$$\to 2\pi u(x) = \pi\cos\phi + a_1 \pi\cos\phi + b_1 \pi\sin\phi$$

$u(1, \phi) = \cos\phi$代入 $\to 2\pi\cos\phi = \pi\cos\phi + a_1 \pi\cos\phi + b_1 \pi\sin\phi$

$\to a_1 = 1,\ b_1 = 0,\ t(1, \theta) = \cos\theta$

$$2\pi u(x) = \int_B -T^E(s,x)\, u(s)\, dB(s) - \int_B U^E(s,x)\, t(s)\, dB(s)$$

$$\to 2\pi u(x) = \int_0^{2\pi} -\left\{-\sum_{m=1}^{\infty} \frac{R^{m-1}}{\rho^m} \cos m(\theta-\phi)\right\} \cos\theta\, d\theta$$
$$- \int_0^{2\pi} \left\{\ln\rho - \sum_{m=1}^{\infty} \frac{1}{m}\left(\frac{R}{\rho}\right)^m \cos m(\theta-\phi)\right\} \cos\theta\, d\theta$$

$$\to 2\pi u(x) = \int_0^{2\pi} -\left\{-\sum_{m=1}^{\infty} \frac{1}{\rho^m} \cos m(\theta-\phi)\right\} \cos\theta\, d\theta$$
$$- \int_0^{2\pi} \left\{\ln\rho - \sum_{m=1}^{\infty} \frac{1}{m} \frac{1}{\rho^m} \cos m(\theta-\phi)\right\} \cos\theta\, d\theta$$

$$\to 2\pi u(x) = 2\int_0^{2\pi} \frac{1}{\rho} (\cos\theta\cos\phi\cos\theta + \sin\theta\sin\phi\sin\theta)\, d\theta$$

$$\rightarrow 2\,\pi \mathbf{u}\,(\mathbf{x}) = 2\left(\frac{1}{\rho}\,\pi\cos\phi\right) \rightarrow \mathbf{u}\,(\mathbf{x}) = \mathbf{u}\,(\rho,\ \phi) = \frac{1}{\rho}\,\cos\phi$$