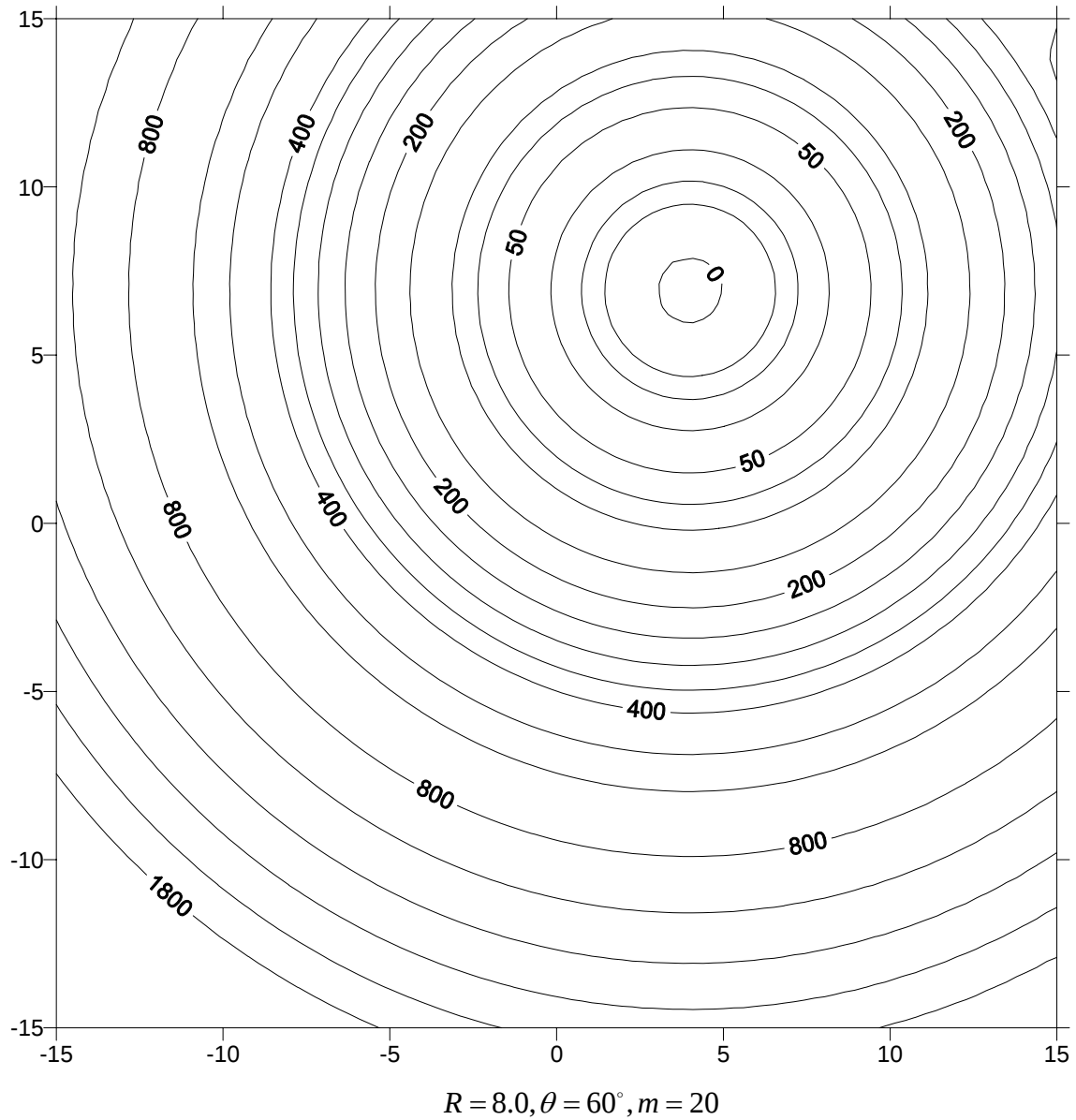


國立台灣海洋大學河海工程研究所 BEM2004 第 11 次作業

1. Please plot the degenerate kernel of the closed-form fundamental solution.

Closed-form :

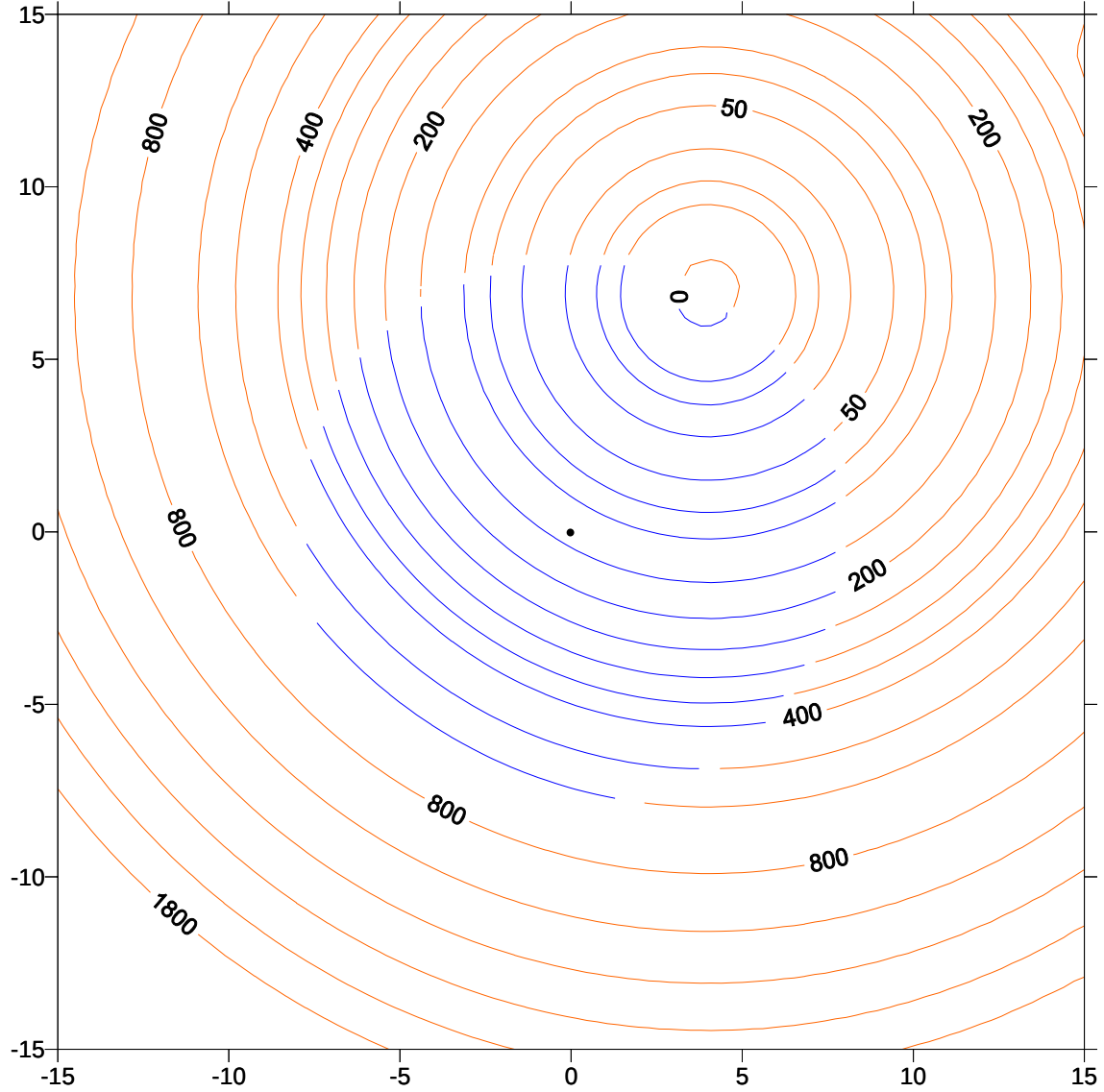
$$U(s, x) = r^2 \ln r, r = |x - s|, \text{ where } x = (\rho, \phi) \text{ and } s = (R, \theta).$$



Degenerate form :

$$U(s, x) = \begin{cases} U^I(s, x) = \rho^2(1 + \ln R) + R^2 \ln R - R\rho(1 + 2\ln R)\cos(\theta - \phi) \\ \quad - \sum_{m=1}^{\infty} \frac{1}{m(m+1)} \frac{\rho^{m+2}}{R^m} \cos[m(\theta - \phi)] + \sum_{m=2}^{\infty} \frac{1}{m(m-1)} \frac{\rho^m}{R^{m-2}} \cos[m(\theta - \phi)], R > \rho \\ U^E(s, x) = R^2(1 + \ln \rho) + \rho^2 \ln \rho - \rho R(1 + 2\ln \rho)\cos(\theta - \phi) \\ \quad - \sum_{m=1}^{\infty} \frac{1}{m(m+1)} \frac{R^{m+2}}{\rho^m} \cos[m(\theta - \phi)] + \sum_{m=2}^{\infty} \frac{1}{m(m-1)} \frac{R^m}{\rho^{m-2}} \cos[m(\theta - \phi)], \rho > R \end{cases}$$

where $x = (\rho, \phi)$ and $s = (R, \theta)$.



$R = 8.0, \theta = 60^\circ, m = 20$

