BEM12-2004s

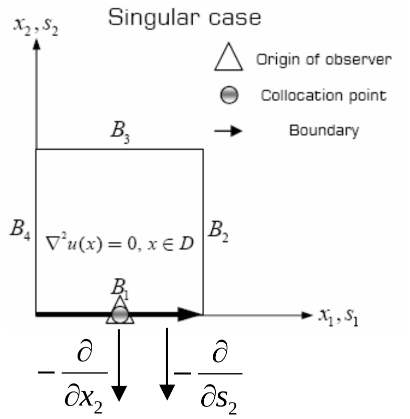
$(x_1, x_2) \Rightarrow (\rho, \phi)$	$(s_1, s_2) \Rightarrow (R, \theta)$
$x_1 = \rho\cos(\phi)$ $x_2 = \rho\sin(\phi)$ $\rho = \sqrt{x_1^2 + x_2^2}$ $\phi = \tan^{-1}(\frac{x_2}{x_1})$	$s_1 = R\cos(\theta)$ $s_2 = R\sin(\theta)$ $R = \sqrt{s_1^2 + s_2^2}$ $\theta = \tan^{-1}(\frac{s_2}{s_1})$
$\frac{\partial\rho}{\partial x_2} = \frac{1}{2}\frac{2x_2}{\sqrt{x_1^2 + x_2^2}} = \frac{\rho\sin(\phi)}{\rho} = \sin(\phi)$ $\frac{\partial\phi}{\partial x_2} = \frac{1/x_1}{1+(x_2/x_1)^2} = \frac{x_1}{x_1^2 + x_2^2} = \frac{\cos(\phi)}{\rho}$	$\frac{\partial R}{\partial s_2} = \frac{1}{2}\frac{2s_2}{\sqrt{s_1^2 + s_2^2}} = \frac{R\sin(\theta)}{R} = \sin(\theta)$ $\frac{\partial\theta}{\partial s_2} = \frac{1/s_1}{1+(s_2/s_1)^2} = \frac{s_1}{s_1^2 + s_2^2} = \frac{\cos(\theta)}{R}$
$\frac{\partial}{\partial n_x}(U)$ $= -\frac{\partial}{\partial x_2}(U)$ $= -[\frac{\partial}{\partial\rho}(U)\frac{\partial\rho}{\partial x_2} + \frac{\partial}{\partial\phi}(U)\frac{\partial\phi}{\partial x_2}]$ $= -[\frac{\partial}{\partial\rho}(U)\sin(\phi) + \frac{\partial}{\partial\phi}(U)\frac{\cos(\phi)}{\rho}]$ $= -[\sin(\phi)\frac{\partial}{\partial\rho}(U) + \frac{\cos(\phi)}{\rho}\frac{\partial}{\partial\phi}(U)]$	$\frac{\partial}{\partial s_x}(U)$ $= -\frac{\partial}{\partial s_2}(U)$ $= -[\frac{\partial}{\partial R}(U)\frac{\partial R}{\partial s_2} + \frac{\partial}{\partial\theta}(U)\frac{\partial\theta}{\partial s_2}]$ $= -[\frac{\partial}{\partial R}(U)\sin(\theta) + \frac{\partial}{\partial\theta}(U)\frac{\cos(\theta)}{R}]$ $= -[\sin(\theta)\frac{\partial}{\partial R}(U) + \frac{\cos(\theta)}{R}\frac{\partial}{\partial\theta}(U)]$

Since $(\rho, \phi) = (0,0)$

$$U = U^i(s,x) = \ln(R) - \sum_{m=1}^{\infty}\frac{1}{m}(\frac{\rho}{R})^m\cos(m(\theta-\phi)) \quad R > \rho$$

$$T = -\frac{\partial}{\partial s_2}(U)$$

$$= -[\sin(\theta)\frac{\partial}{\partial R}(U) + \frac{\cos(\theta)}{R}\frac{\partial}{\partial\theta}(U)]$$



$$= -[\sin(\theta)(\frac{1}{R} - \sum_{m=1}^{\infty} \frac{-m}{m}\frac{\rho^m}{R^{m+1}}\cos(m(\theta-\phi)) + \frac{\cos(\theta)}{R}(-\sum_{m=1}^{\infty}\frac{1}{m}(\frac{\rho}{R})^m (m)(-\sin(m(\theta-\phi))))]$$

$$= -\frac{\sin(\theta)}{R} - \sum_{m=1}^{\infty}\frac{\rho^m}{R^{m+1}}\sin(\theta)\cos(m(\theta-\phi)) - \sum_{m=1}^{\infty}\frac{\rho^m}{R^{m+1}}\cos(\theta)\sin(m(\theta-\phi))$$

$$= -\frac{\sin(\theta)}{R} - \sum_{m=1}^{\infty}\frac{\rho^m}{R^{m+1}}\sin[(m+1)(\theta) - m\phi)]$$

$$M = -\frac{\partial}{\partial x_2}(T)$$

$$= -[\sin(\phi)\frac{\partial}{\partial\rho}(T) + \frac{\cos(\phi)}{\rho}\frac{\partial}{\partial\phi}(T)]$$

$$= -[\sin(\phi)(-\sum_{m=1}^{\infty}\frac{m\rho^{m-1}}{R^{m+1}}\sin[(m+1)(\theta) - m\phi)]) + \frac{\cos(\phi)}{\rho}(-\sum_{m=1}^{\infty}\frac{\rho^m}{R^{m+1}}(-m)\cos[(m+1)(\theta) - m\phi)])]$$

$(\rho,\phi) = (0,0)$ 代入

$$M = -(-\frac{1}{R^2}(-1)\cos(2\theta)) = -\frac{1}{R^2}\cos(2\theta)$$

When $\theta = \pi$

$$M = -\frac{1}{R^2}\cos(2\pi) = -\frac{1}{R^2}$$

When $\theta = 0$

$$M = -\frac{1}{R^2}\cos(0) = -\frac{1}{R^2}$$

$$-\int_{0.5}^{\varepsilon} -\frac{1}{R^2}dR + \int_{\varepsilon}^{0.5} -\frac{1}{R^2}dR$$

$$= -\frac{1}{\varepsilon} + \frac{1}{0.5} + \frac{1}{0.5} - \frac{1}{\varepsilon}$$

$$= 4 - \frac{2}{\varepsilon}$$