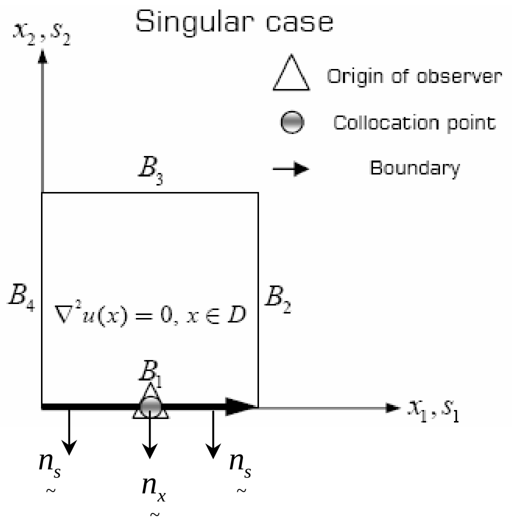
BEM12-2004s

When $\theta=\pi$, $\underset{\sim}{n_s}=0\hat{e}_R+1\hat{e}_\theta$

When $\theta=0$, $\underset{\sim}{n_s}=0\hat{e}_R-1\hat{e}_\theta$



$$\hat{i}=\cos\phi\,\hat{e}_\rho-\sin\phi\,\hat{e}_\phi$$

$$\hat{j}=\sin\phi\,\hat{e}_\rho+\cos\phi\,\hat{e}_\phi$$

$$\underset{\sim}{n_x}=0\hat{i}-1\hat{j}$$

$$=0\cdot(\cos\phi\,\hat{e}_\rho-\sin\phi\,\hat{e}_\phi)-1\cdot(\sin\phi\,\hat{e}_\rho+\cos\phi\,\hat{e}_\phi)$$

$$=-\sin\phi\,\hat{e}_\rho-\cos\phi\,\hat{e}_\phi$$

$$U=U^i(s,x)=\ln(R)-\sum_{m=1}^{\infty}\frac{1}{m}\left(\frac{\rho}{R}\right)^m\cos(m(\theta-\phi))\quad R>\rho$$

When $\theta=\pi$,

$$T=\frac{\partial U}{\partial n_s}=\nabla U\cdot\underset{\sim}{n_s}=\left(\frac{\partial U}{\partial R}\hat{e}_R+\frac{1}{R}\frac{\partial U}{\partial\theta}\hat{e}_\theta\right)\cdot(0\hat{e}_R+1\hat{e}_\theta)=\sum_{m=1}^{\infty}\frac{\rho^m}{R^{m+1}}\sin(m(\theta-\phi))$$

When $\theta=0$,

$$T=\frac{\partial U}{\partial n_s}=\nabla U\cdot\underset{\sim}{n_s}=\left(\frac{\partial U}{\partial R}\hat{e}_R+\frac{1}{R}\frac{\partial U}{\partial\theta}\hat{e}_\theta\right)\cdot(0\hat{e}_R-1\hat{e}_\theta)=-\sum_{m=1}^{\infty}\frac{\rho^m}{R^{m+1}}\sin(m(\theta-\phi))$$

$$L=\frac{\partial U}{\partial n_x}=\nabla U\cdot\underset{\sim}{n_x}=\left(\frac{\partial U}{\partial\rho}\hat{e}_\rho+\frac{1}{\rho}\frac{\partial U}{\partial\phi}\hat{e}_\phi\right)\cdot(-\sin\phi\,\hat{e}_\rho-\cos\phi\,\hat{e}_\phi)=\sum_{m=1}^{\infty}\frac{\rho^{m-1}}{R^m}\sin(m(\theta-\phi))\cos\phi$$

When $\theta=\pi$,

$$M=\frac{\partial^2 U}{\partial n_x\partial n_s}=\nabla L\cdot\underset{\sim}{n_s}=\left(\frac{\partial L}{\partial R}\hat{e}_R+\frac{1}{R}\frac{\partial L}{\partial\theta}\hat{e}_\theta\right)\cdot(0\hat{e}_R+1\hat{e}_\theta)=\sum_{m=1}^{\infty}\frac{\rho^{m-1}}{R^{m+1}}m\cos(m(\theta-\phi))\cos\phi$$

When $\theta=0$,

$$M=\frac{\partial^2 U}{\partial n_x\partial n_s}=\nabla L\cdot\underset{\sim}{n_s}=\left(\frac{\partial L}{\partial R}\hat{e}_R+\frac{1}{R}\frac{\partial L}{\partial\theta}\hat{e}_\theta\right)\cdot(0\hat{e}_R-1\hat{e}_\theta)=-\sum_{m=1}^{\infty}\frac{\rho^{m-1}}{R^{m+1}}m\cos(m(\theta-\phi))\cos\phi$$

$\rho = 0$ 代入

When $\theta = \pi$, $M = \dfrac{1}{R^2}\cos(\pi - \phi)\cos\phi = -\dfrac{1}{R^2}\cos^2\phi$

When $\theta = 0$, $M = -\dfrac{1}{R^2}\cos(-\phi)\cos\phi = -\dfrac{1}{R^2}\cos^2\phi$

When $\phi = \pi$, $M = -\dfrac{1}{R^2}$

When $\phi = 0$, $M = -\dfrac{1}{R^2}$

$$-\int_{0.5}^{\varepsilon} -\frac{1}{R^2}\,dR + \int_{\varepsilon}^{0.5} -\frac{1}{R^2}\,dR$$
$$= -\frac{1}{\varepsilon} + \frac{1}{0.5} + \frac{1}{0.5} - \frac{1}{\varepsilon}$$
$$= 4 - \frac{2}{\varepsilon}$$