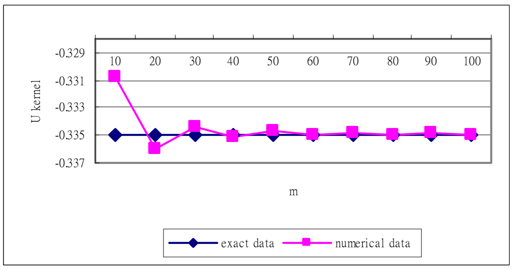
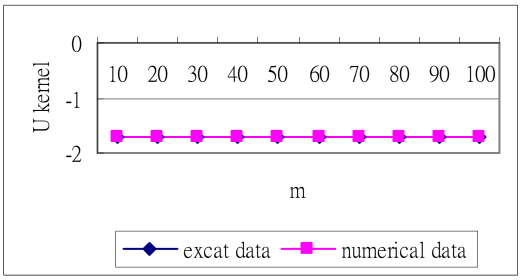
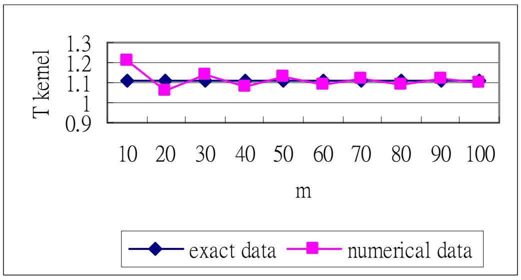
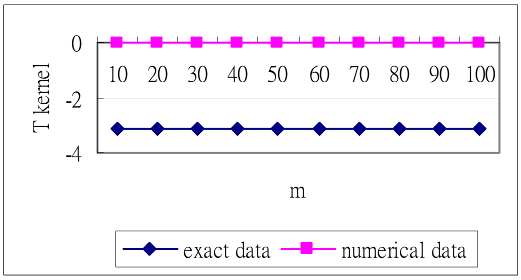
Solution of BEMHW # 12 Dec, 21, 2004

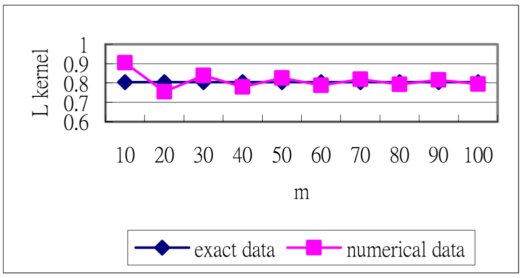
Regular case	Singular case

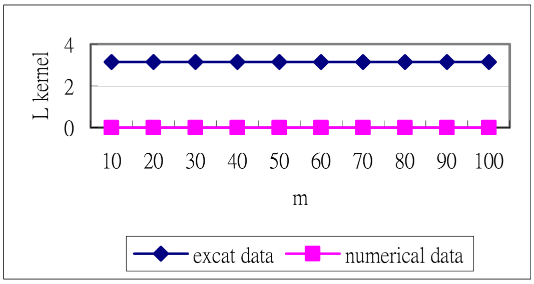


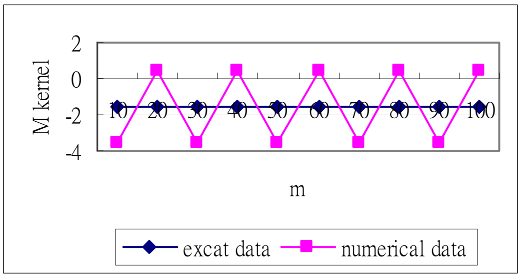


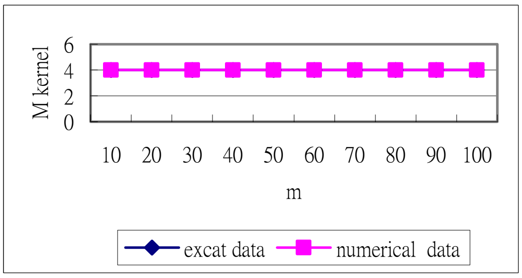












$$\mathbf{UI} = \mathrm{Log}[\mathbf{R}] - \sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{\rho}{\mathbf{R}}\right)^{m} \mathrm{Cos}[m\,(\theta - \phi)];$$

$$\mathbf{UE} = \mathrm{Log}[\rho] - \sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{\mathbf{R}}{\rho}\right)^{m} \mathrm{Cos}[m\,(\theta - \phi)];$$

$$\mathbf{TI} = \sum_{m=1}^{\infty} \left(\frac{\rho^{m}}{\mathbf{R}^{m+1}}\right) \mathrm{Sin}[m\,(\theta - \phi)];$$

$$\mathbf{TE} = \sum_{m=1}^{\infty} \left(\frac{\mathbf{R}^{m-1}}{\rho^{m}}\right) \mathrm{Sin}[m\,(\theta - \phi)];$$

$$\mathbf{LI} = \sum_{m=1}^{\infty} \left(\frac{\rho^{m-1}}{\mathbf{R}^{m}}\right) \mathrm{Sin}[m\,(\theta - \phi)];$$

$$\mathbf{LE} = \sum_{m=1}^{\infty} \left(\frac{\mathbf{R}^{m}}{\rho^{m+1}}\right) \mathrm{Sin}[m\,(\theta - \phi)];$$

$$\mathbf{MI} = \sum_{m=1}^{\infty} m \left(\frac{\rho^{m-1}}{\mathbf{R}^{m+1}}\right) \mathrm{Cos}[m\,(\theta - \phi)];$$

$$\mathbf{ME} = \sum_{m=1}^{\infty} m \left(\frac{\mathbf{R}^{m-1}}{\rho^{m+1}}\right) \mathrm{Cos}[m\,(\theta - \phi)];$$

Regular Case

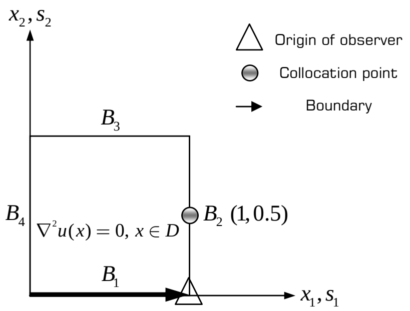
$$\theta = \pi;\ \phi = \frac{\pi}{2};\ \rho = 0.5;$$

$$\mathbf{U} = -\left(\int_{0.5}^{0} \mathbf{UE}\, d\mathbf{R} + \int_{1}^{0.5} \mathbf{UI}\, d\mathbf{R}\right)$$

$$\mathbf{T} = -\left(\int_{0.5}^{0} \mathbf{TE}\, d\mathbf{R} + \int_{1}^{0.5} \mathbf{TI}\, d\mathbf{R}\right)$$

$$\mathbf{L} = -\left(\int_{0.5}^{0} \mathbf{LE}\, d\mathbf{R} + \int_{1}^{0.5} \mathbf{LI}\, d\mathbf{R}\right)$$

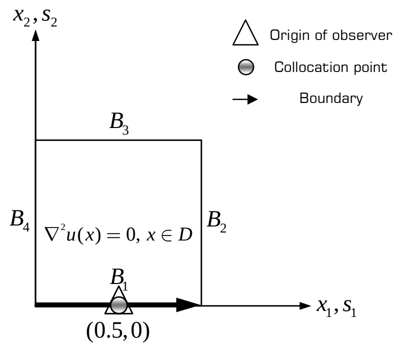
$$\mathbf{M} = -\left(\int_{0.5}^{0} \mathbf{ME}\, d\mathbf{R} + \int_{1}^{0.5} \mathbf{MI}\, d\mathbf{R}\right)$$



	U	T	L	M
m=50	-0.334662	1.12714	0.824719	-3.6
m=100	-0.334903	1.09715	0.794719	0.4
Exact	-0.335	1.10715	0.80472	-1.6

U	T	L	M
$\begin{bmatrix} \times & \times & \times & \times \\ -0.335 & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{bmatrix}$	$\begin{bmatrix} \times & \times & \times & \times \\ 1.10715 & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{bmatrix}$	$\begin{bmatrix} \times & \times & \times & \times \\ 0.80472 & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{bmatrix}$	$\begin{bmatrix} \times & \times & \times & \times \\ -1.6 & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{bmatrix}$

Singular Case



$\theta = \pi\,,\ \phi = 0\,,\ \rho = 0$

$$\mathbf{U1} = -\int_{0.5}^{\epsilon} \mathbf{UI}\, d\mathbf{R};$$

$$\mathbf{T1} = -\int_{0.5}^{\epsilon} \mathbf{TI}\, d\mathbf{R};$$

$$\mathbf{L1} = -\int_{0.5}^{\epsilon} \mathbf{LI}\, d\mathbf{R};$$

$$\mathbf{M1} = -\int_{0.5}^{\epsilon} \mathbf{MI}\, d\mathbf{R};$$

$$U = \lim[U1 + U2, \varepsilon \to 0]$$

$$T = \lim[T1 + T2, \varepsilon \to 0]$$

$\theta = 0\,,\ \phi = 0\,,\ \rho = 0$

$$\mathbf{U2} = \int_{\epsilon}^{0.5} \mathbf{UI}\, d\mathbf{R};$$

$$\mathbf{T2} = \int_{\epsilon}^{0.5} \mathbf{TI}\, d\mathbf{R};$$

$$\mathbf{L2} = \int_{\epsilon}^{0.5} \mathbf{LI}\, d\mathbf{R};$$

$$\mathbf{M2} = -\int_{\epsilon}^{0.5} \mathbf{MI}\, d\mathbf{R}$$

$$L = \lim[L1 + L2, \varepsilon \to 0]$$

$$M = \lim[M1 + M2 + \frac{2}{\varepsilon}, \varepsilon \to 0]$$

	U	T	L	M
m=50	-1.69315	0^*	0^*	4
m=100	-1.69315	0^*	0^*	4
Exact	-1.693	$-\pi$	π	4

* free of bump contribution.

U	T	L	M
$\begin{bmatrix} -1.693 & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{bmatrix}$	$\begin{bmatrix} -\pi & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{bmatrix}$	$\begin{bmatrix} \pi & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{bmatrix}$	$\begin{bmatrix} 4 & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{bmatrix}$