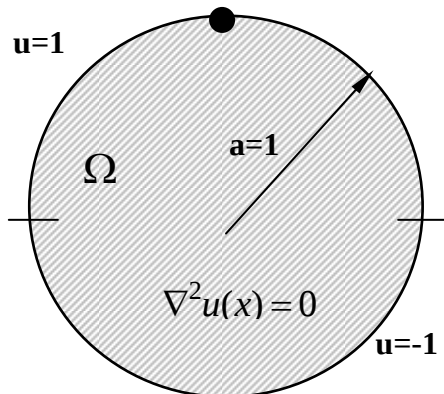
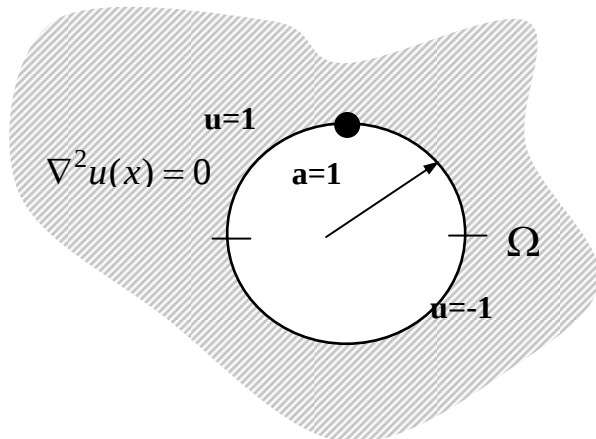


Problem statement and solution



Exact solution: $\frac{2}{\pi} \tan^{-1} \left(\frac{-2y}{1-x^2-y^2} \right)$

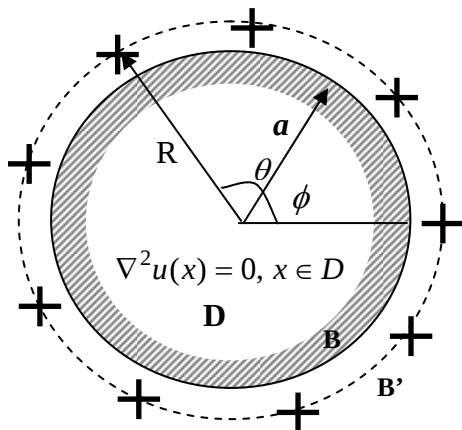
Interior problem



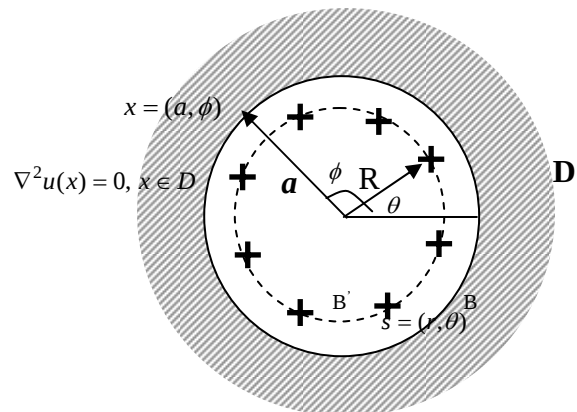
Exact solution: $\frac{2}{\pi} \tan^{-1} \left(\frac{2y}{x^2+y^2-1} \right)$

Exterior problem

Distribution of source point for the indirect method(MFS)

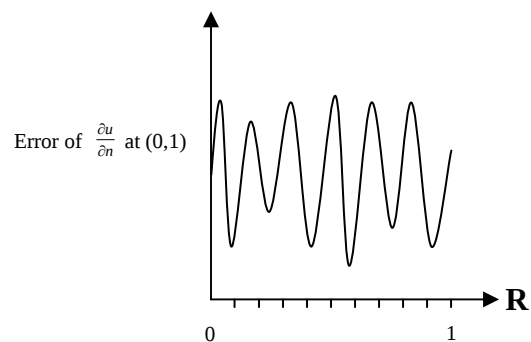
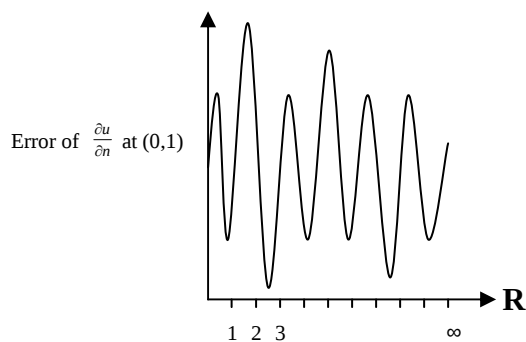


+ : distribution of source point

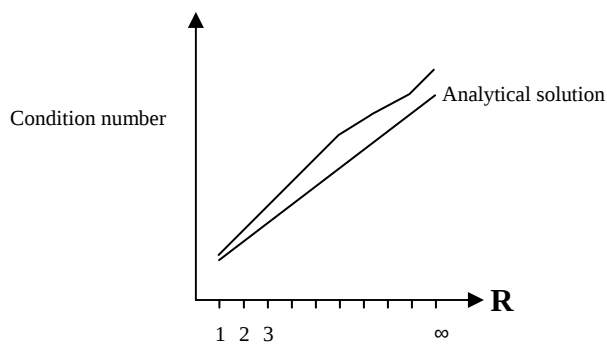


+ : distribution of source point

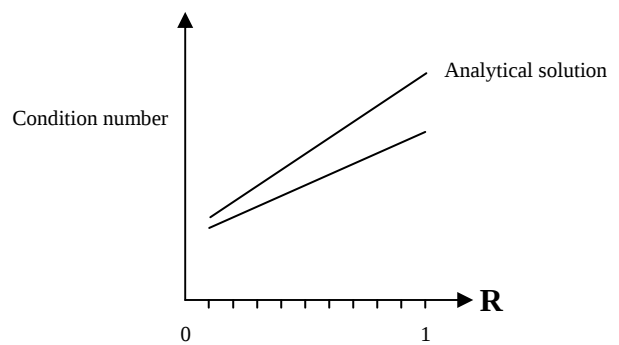
Error plot by using indirect method (21 source are distributed)



Plot condition number for 21 points



Interior case



Exterior case

Calculate the errors of t of Trefftz method and MFS

	Error of t (percent)
Trefftz method (21 bases)	
MFS (21 points)	

Trefftz method :

$$u(x) = \sum_{j=1}^{N_T} a_j T_j(x)$$

where N_T is the number of complete functions.

For the interior problem, we choose $T = \{1, \rho^n \cos(n\phi), \rho^n \sin(n\phi)\}$, $n = 1, 2, 3, \dots$

For the exterior problem, we choose $T = \{\ln \rho, \rho^{-n} \cos(n\phi), \rho^{-n} \sin(n\phi)\}$, $n = 1, 2, 3, \dots$

MFS :

$$u(x) = \sum_{j=1}^{N_F} b_j U(x, s_j)$$

where $U(x, s) = \ln \left| \frac{x - s}{\bar{x} - \bar{s}} \right|$, N_F is the number of sources.