BEM H.W .003 M93520010 陳柏源

$$\frac{d^4 U(x, s)}{d x^4} + q^4 U(x, s) = \delta(x - s)$$

(1) single pole ($q = 0$)

(2) double pole ($q \neq 0$)

ANS

(1)

$$V(k) = \int_{-\infty}^{\infty} v(x)\, e^{-i k x}\, dx$$

$$\rightarrow \int_{-\infty}^{\infty} v'(x)\, e^{-i k x}\, dx = i k * V(k)$$

$$\rightarrow \int_{-\infty}^{\infty} v^{(4)}(x)\, e^{-i k x}\, dx = (i k)^4 * V(k)$$

→ Inverse Fourier Transform

$$\rightarrow v(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} V(x)\, e^{i k x}\, dx$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-i k x}}{k^4}\, e^{i k x}\, dx$$

→ Single pole ($q = 0$)

→ $k = 0, 0, 0, 0$ ☹

(2)

$$v(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-i k x}}{k^4 + q^4}\, e^{i k x}\, dx$$

$$\rightarrow k = q\, e^{i * \frac{1}{4} * \pi},\ q\, e^{i * \frac{3}{4} * \pi},\ q\, e^{i \frac{5}{4} \pi},\ q\, e^{i \frac{7}{4} \pi}$$

(a) $x > s$

$$\frac{1}{2\pi} * 2\pi i \left(\text{Res}\left(k = q\, e^{i * \frac{1}{4} * \pi}\right) + \text{Res}\left(k = q\, e^{i * \frac{3}{4} * \pi}\right)\right)$$

$$= i * \left(\text{Limit}\left[\frac{e^{i k (x-s)}}{\left(k - q\, e^{i \frac{3}{4} \pi}\right)\left(k - q\, e^{i \frac{5}{4} \pi}\right)\left(k - q\, e^{i \frac{7}{4} \pi}\right)}, k \rightarrow q\, e^{\frac{i \pi}{4}}\right] + \text{Limit}\left[\frac{e^{i k (x-s)}}{\left(\left(k - q\, e^{\frac{i \pi}{4}}\right)\left(k - q\, e^{i \frac{5}{4} \pi}\right)\left(k - q\, e^{i \frac{7}{4} \pi}\right)\right)}, k \rightarrow q\, e^{\frac{i 3 \pi}{4}}\right]\right)$$

$$= i \left(-\frac{(-1)^{1/4}\, e^{(-1)^{3/4} q (-s+x)}}{4 q^3} + -\frac{(-1)^{3/4}\, e^{(-1)^{1/4} q (s-x)}}{4 q^3}\right)$$

$$= \frac{(-1)^{1/4} \left(e^{(-1)^{1/4} q (s-x)} - i\, e^{(-1)^{3/4} q (-s+x)}\right)}{4 q^3}$$

$$\to \frac{(-1)^{1/4}\left(e^{(-1)^{1/4} q (s-x)} - i\, e^{(-1)^{3/4} q (-s+x)}\right)}{4 q^3} - \frac{?}{4 q^3}$$

$$\to \frac{0}{0}$$

→ L'Hospital

$$\to \text{Limit}\left[\frac{(-1)^{1/4}\left((-1)^{1/4} e^{(-1)^{1/4} q (s-x)} (s-x) + (-1)^{1/4} e^{(-1)^{3/4} q (-s+x)} (-s+x)\right)}{12 q^2}, q \to 0\right]$$

$$\to \text{Limit}\left[\frac{(-1)^{1/4}\left(i\, e^{(-1)^{1/4} q (s-x)} (s-x)^2 - e^{(-1)^{3/4} q (-s+x)} (-s+x)^2\right)}{24 q}, q \to 0\right]$$

$$\to \text{Limit}\left[\frac{(-1)^{1/4}\left((-1)^{3/4} e^{(-1)^{1/4} q (s-x)} (s-x)^3 - (-1)^{3/4} e^{(-1)^{3/4} q (-s+x)} (-s+x)^3\right)}{24}, q \to 0\right]$$

$$\to -\frac{1}{12}(s-x)^3$$

$$\to \frac{1}{12}(x-s)^3 \qquad \text{☺}$$

(b) $x \prec s$

$$-\frac{1}{2\pi} * 2\pi i \left(\text{Res}\left(k = q\, e^{i*\frac{5}{4}*\pi}\right) + \text{Res}\left(k = q\, e^{i*\frac{7}{4}*\pi}\right)\right)$$

$$= -i * \left(\text{Limit}\left[\frac{e^{i k (x-s)}}{\left(k - q\, e^{i\frac{1}{4}\pi}\right)\left(k - q\, e^{i\frac{3}{4}\pi}\right)\left(k - q\, e^{i\frac{7}{4}\pi}\right)}, k \to q\, e^{\frac{i 5\pi}{4}}\right] + \text{Limit}\left[\frac{e^{i k (x-s)}}{\left(\left(k - q\, e^{\frac{i\pi}{4}}\right)\left(k - q\, e^{i\frac{3}{4}\pi}\right)\left(k - q\, e^{i\frac{5}{4}\pi}\right)\right)}, k \to q\, e^{\frac{i 7\pi}{4}}\right]\right)$$

$$= -i * \left(\frac{(-1)^{1/4} e^{(-1)^{3/4} q (s-x)}}{4 q^3} + \frac{(-1)^{3/4} e^{(-1)^{1/4} q (-s+x)}}{4 q^3}\right)$$

$$= -\frac{(-1)^{3/4}\left(e^{(-1)^{3/4} q (s-x)} + i\, e^{(-1)^{1/4} q (-s+x)}\right)}{4 q^3} - \frac{?}{4 q^3}$$

$$\to \frac{0}{0}$$

→ L'Hospital

$$\to \text{Limit}\left[-\frac{(-1)^{3/4}\left((-1)^{3/4} e^{(-1)^{3/4} q (s-x)} (s-x) + (-1)^{3/4} e^{(-1)^{1/4} q (-s+x)} (-s+x)\right)}{12 q^2}, q \to 0\right]$$

$$\to \text{Limit}\left[-\frac{(-1)^{3/4}\left(-i\, e^{(-1)^{3/4} q (s-x)} (s-x)^2 - e^{(-1)^{1/4} q (-s+x)} (-s+x)^2\right)}{24 q}, q \to 0\right]$$

$$\to \text{Limit}\left[-\frac{(-1)^{3/4}\left((-1)^{1/4} e^{(-1)^{3/4} q (s-x)} (s-x)^3 - (-1)^{1/4} e^{(-1)^{1/4} q (-s+x)} (-s+x)^3\right)}{24}, q \to 0\right]$$

$$\to \frac{1}{12}(s-x)^3$$

$$= -\frac{1}{12}(x-s)^3 \qquad \text{☺}$$

$$\nu(x) = \begin{cases} \frac{1}{12}(x-s)^3, & x \succ s \\ -\frac{1}{12}(x-s)^3, & x \prec s \end{cases}$$

Null