BEM H.W .014 M93520010

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$$U^i = \ln[R] - \sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{\rho}{R}\right)^m \text{Cos}[m(\theta-\phi)],\ R > \rho$$

$$U^e = \ln[\rho] - \sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{R}{\rho}\right)^m \text{Cos}[m(\theta-\phi)],\ R < \rho$$

$$T^i = \frac{1}{R} + \sum_{m=1}^{\infty} \frac{\rho^m}{R^{m+1}} \text{Cos}[m(\theta-\phi)],\ R > \rho$$

$$T^e = -\sum_{m=1}^{\infty} \frac{R^{m-1}}{\rho^m} \text{Cos}[m(\theta-\phi)],\ R < \rho$$

$$L^i = -\sum_{m=1}^{\infty} \frac{\rho^{m-1}}{R^m} \text{Cos}[m(\theta-\phi)],\ R > \rho$$

$$L^e = \frac{1}{\rho} + \sum_{m=1}^{\infty} \frac{R^m}{\rho^{m+1}} \text{Cos}[m(\theta-\phi)],\ R < \rho$$

$$M^i = \sum_{m=1}^{\infty} \frac{\rho^{m-1}}{R^{m+1}}\, m\, \text{Cos}[m(\theta-\phi)],\ R > \rho$$

$$M^e = \sum_{m=1}^{\infty} \frac{R^{m-1}}{\rho^{m+1}}\, m\, \text{Cos}[m(\theta-\phi)],\ R < \rho$$

ANS.

- $\int_0^{2\pi} M(\rho, \phi; \bar{\rho}, \bar{\phi}) \int_0^{2\pi} M(R, \theta; \rho, \phi) * \text{Sin}[n\theta]\, d\theta\, d\phi$

(i)

$$\int_0^{2\pi} M(R, \theta; \rho, \phi)\, d\theta$$

$$= \int_0^{2\pi} \left(\sum_{m=1}^{\infty} \frac{R^{m-1}}{\rho^{m+1}}\, m\, \text{Cos}[m(\theta-\phi)]\right) * \text{Sin}[n\theta]\, d\theta$$

$$= \int_0^{2\pi} \left(\sum_{m=1}^{\infty} \frac{R^{m-1}}{\rho^{m+1}}\, m\, (\text{Cos}[m\theta]\, \text{Cos}[m\phi] + \text{Sin}[m\theta]\, \text{Sin}[m\phi])\right) * \text{Sin}[n\theta]\, d\theta$$

$$= \int_0^{2\pi} \frac{R^{n-1}}{\rho^{n+1}}\, n\, \text{Sin}^2[n\theta]\, \text{Sin}[n\phi]\, d\theta$$

$$= \frac{R^{n-1}}{\rho^{n+1}}\, n \left(\frac{\theta}{2} - \frac{\text{Sin}[2n\theta]}{4n}\right) \text{Sin}[n\phi] \Big|_0^{2\pi}$$

$$= \frac{R^{n-1}}{\rho^{n+1}}\, n\pi\, \text{Sin}[n\phi]$$

(ii)

$$\int_0^{2\pi} M(\rho, \phi; \bar{\rho}, \bar{\phi}) \int_0^{2\pi} M(R, \theta; \rho, \phi) * \text{Sin}[n\theta]\, d\theta\, d\phi$$

$$= \int_0^{2\pi} \left(\sum_{m=1}^{\infty} \frac{\rho^{m-1}}{(\bar{\rho})^{m+1}}\, m\, \text{Cos}[m(\phi-\bar{\phi})]\right) * \left(\frac{R^{n-1}}{\rho^{n+1}}\, n\pi\, \text{Sin}[n\phi]\right) d\phi$$

$$= \int_0^{2\pi} \left(\sum_{m=1}^{\infty} \frac{\rho^{m-1}}{(\bar{\rho})^{m+1}} m \left(\text{Cos}[m\,\phi]\, \text{Cos}[m\,\bar{\phi}] + \text{Sin}[m\,\phi]\, \text{Sin}[m\,\bar{\phi}] \right) \right) * \left(\frac{R^{n-1}}{\rho^{n+1}} n\,\pi\, \text{Sin}[n\,\phi] \right) d\phi$$

$$= \int_0^{2\pi} \frac{R^{n-1}}{(\bar{\rho})^{n+1}\,\rho^2} n^2\,\pi\, \text{Sin}^2[n\,\phi]\, \text{Sin}[n\,\bar{\phi}]\, d\phi$$

$$= \frac{R^{n-1}}{(\bar{\rho})^{n+1}\,\rho^2} n^2\,\pi \left(\frac{\theta}{2} - \frac{\text{Sin}[2\,n\,\theta]}{4\,n} \right) \text{Sin}[n\,\bar{\phi}] \Big|_{0}^{2\pi}$$

$$= \frac{R^{n-1}}{(\bar{\rho})^{n+1}\,\rho^2} n^2\,\pi^2\, \text{Sin}[n\,\bar{\phi}]$$

$\Longrightarrow (\rho = \bar{\rho} = R = 1)$

$$= \boxed{n^2\,\pi^2\, \text{Sin}[n\,\bar{\phi}]}$$

- $\int_0^{2\pi} U(\rho, \phi; \bar{\rho}, \bar{\phi}) \int_0^{2\pi} U(R, \theta; \rho, \phi) * \text{Cos}[n\,\theta]\, d\theta\, d\phi$

(i)

$$\int_0^{2\pi} U(R, \theta; \rho, \phi)\, d\theta$$

$$= \int_0^{2\pi} \left(\ln[\rho] - \sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{R}{\rho}\right)^m \text{Cos}[m\,(\theta - \phi)] \right) * \text{Cos}[n\,\theta]\, d\theta$$

$$= \int_0^{2\pi} \left(\ln[\rho] * \text{Cos}[n\,\theta] - \left(\sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{R}{\rho}\right)^m \text{Cos}[m\,(\theta - \phi)] \right) * \text{Cos}[n\,\theta] \right) d\theta$$

$$= \int_0^{2\pi} \left(\ln[\rho] * \text{Cos}[n\,\theta] - \left(\sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{R}{\rho}\right)^m (\text{Cos}[m\,\theta]\, \text{Cos}[m\,\phi] + \text{Sin}[m\,\theta]\, \text{Sin}[m\phi]) \right) * \text{Cos}[n\,\theta] \right) d\theta$$

$$= \int_0^{2\pi} \left(\ln[\rho] * \text{Cos}[n\,\theta] - \left(\frac{1}{n} \left(\frac{R}{\rho}\right)^n \text{Cos}^2[n\,\theta]\, \text{Cos}[n\,\phi] \right) \right) d\theta$$

$$= \ln[\rho] * \frac{1}{n} * \text{Sin}[n\,\theta] \Big|_{0}^{2\pi} - \frac{1}{n} \left(\frac{R}{\rho}\right)^n \left(\frac{\theta}{2} + \frac{\text{Sin}[2\,n\,\theta]}{4\,n} \right) \text{Cos}[n\,\phi] \Big|_{0}^{2\pi}$$

$$= -\frac{1}{n} \left(\frac{R}{\rho}\right)^n \pi\, \text{Cos}[n\,\phi]$$

(ii)

$$\int_0^{2\pi} U(\rho, \phi; \bar{\rho}, \bar{\phi}) \int_0^{2\pi} U(R, \theta; \rho, \phi) * \text{Cos}[n\,\theta]\, d\theta\, d\phi$$

$$= \int_0^{2\pi} \left(\ln[\bar{\rho}] - \sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{\rho}{\bar{\rho}}\right)^m \text{Cos}[m\,(\phi - \bar{\phi})] \right) * \left(-\frac{1}{n} \left(\frac{R}{\rho}\right)^n \pi\, \text{Cos}[n\,\phi] \right) d\phi$$

$$= \int_0^{2\pi} \Bigg\{ \left(\ln[\bar{\rho}] * \left(-\frac{1}{n} \left(\frac{R}{\rho}\right)^n \pi\, \text{Cos}[n\,\phi] \right) \right) + \left(-\sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{\rho}{\bar{\rho}}\right)^m \text{Cos}[m\,(\phi - \bar{\phi})] \right) * \left(-\frac{1}{n} \left(\frac{R}{\rho}\right)^n \pi\, \text{Cos}[n\,\phi] \right) \Bigg\} d\phi$$

$$= \int_0^{2\pi} \Bigg\{ \left(\ln[\bar{\rho}] * \left(-\frac{1}{n} \left(\frac{R}{\rho}\right)^n \pi\, \text{Cos}[n\,\phi] \right) \right) + \left(-\sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{\rho}{\bar{\rho}}\right)^m \left(\text{Cos}[m\,\phi]\, \text{Cos}[m\,\bar{\phi}] + \text{Sin}[m\,\phi]\, \text{Sin}[m\,\bar{\phi}] \right) \right) * \left(-\frac{1}{n} \left(\frac{R}{\rho}\right)^n \pi\, \text{Cos}[n\,\phi] \right) \Bigg\} d\phi$$

$$= \int_0^{2\pi} \left\{ \ln[\bar{\rho}] * \left(-\frac{1}{n}\right) \left(\frac{R}{\rho}\right)^n \pi \left(\frac{1}{n}\right) \cos[n\phi] + \frac{1}{n^2} \left(\frac{\rho}{\bar{\rho}}\right)^n \pi \cos^2[n\phi] \cos[n\bar{\phi}] \right\} d\phi$$

$$= -\ln[\bar{\rho}] * \left(\frac{R}{\rho}\right)^n \pi \sin[n\phi] \Big|_0^{2\pi} + \frac{1}{n^2} \left(\frac{\rho}{\bar{\rho}}\right)^n \pi \left(\frac{\phi}{2} + \frac{\sin[2n\phi]}{4n}\right) \cos[n\bar{\phi}] \Big|_0^{2\pi}$$

$\Longrightarrow (\rho = \bar{\rho} = R = 1)$

$$= \boxed{\frac{1}{n^2} \pi^2 \cos[n\bar{\phi}]}$$

- $\int_0^{2\pi} T^i (\rho, \phi; \bar{\rho}, \bar{\phi}) \int_0^{2\pi} T^e (R, \theta; \rho, \phi) * \cos[n\theta] \, d\theta \, d\phi$

(i)

$$\int_0^{2\pi} T^e (R, \theta; \rho, \phi) * \cos[n\theta] \, d\theta$$

$$= \int_0^{2\pi} \left(-\sum_{m=1}^{\infty} \frac{R^{m-1}}{\rho^m} \cos[m(\theta - \phi)] \right) * \cos[n\theta] \, d\theta$$

$$= \int_0^{2\pi} \left(-\sum_{m=1}^{\infty} \frac{R^{m-1}}{\rho^m} (\cos[m\theta] \cos[m\phi] + \sin[m\theta] \sin[m\phi]) \right) * \cos[n\theta] \, d\theta$$

$$= \int_0^{2\pi} -\frac{R^{n-1}}{\rho^n} \cos^2[n\theta] \cos[n\phi] \, d\theta$$

$$= -\frac{R^{n-1}}{\rho^n} \left(\frac{\theta}{2} + \frac{\sin[2n\theta]}{4n}\right) \cos[n\phi] \Big|_0^{2\pi}$$

$$= -\frac{R^{n-1}}{\rho^n} \pi \cos[n\phi]$$

(ii)

$$\int_0^{2\pi} T^i (\rho, \phi; \bar{\rho}, \bar{\phi}) \int_0^{2\pi} T^e (R, \theta; \rho, \phi) * \cos[n\theta] \, d\theta \, d\phi$$

$$= \int_0^{2\pi} \frac{1}{\rho} + \sum_{m=1}^{\infty} \frac{(\bar{\rho})^m}{\rho^{m+1}} \cos[m(\phi - \bar{\phi})] * \left(-\frac{R^{n-1}}{\rho^n} \pi \cos[n\phi]\right) d\phi$$

$$= \int_0^{2\pi} \frac{1}{\rho} + \left\{ \sum_{m=1}^{\infty} \frac{(\bar{\rho})^m}{\rho^{m+1}} (\cos[m\phi] \cos[m\bar{\phi}] + \sin[m\phi] \sin[m\bar{\phi}]) \right\} * \left(-\frac{R^{n-1}}{\rho^n} \pi \cos[n\phi]\right) d\phi$$

$$= \int_0^{2\pi} \left(-\frac{R^{n-1}}{\rho^{n+1}} \pi \cos[n\phi] + \frac{(\bar{\rho})^n}{\rho^{n+1}} * \left(-\frac{R^{n-1}}{\rho^n}\right) \pi \cos^2[n\phi] \cos[n\bar{\phi}] \right) d\phi$$

$$= -\frac{R^{n-1}}{\rho^{n+1}} \pi \left(\frac{1}{n}\right) \sin[n\phi] \Big|_0^{2\pi} - \frac{(\bar{\rho})^n}{\rho^{n+1}} \frac{R^{n-1}}{\rho^n} \pi \left(\frac{\phi}{2} + \frac{\sin[2n\phi]}{4n}\right) \cos[n\bar{\phi}] \Big|_0^{2\pi}$$

$$= -\frac{(\bar{\rho})^n R^{n-1}}{\rho^{2n+1}} \pi^2 \cos[n\phi]$$

$\Longrightarrow (\rho = \bar{\rho} = R = 1)$

$$= \boxed{-\pi^2 \cos[n\phi]}$$

- $\int_0^{2\pi} L^i (\rho, \phi; \bar{\rho}, \bar{\phi}) \int_0^{2\pi} L^e (R, \theta; \rho, \phi) * \cos[n\theta] \, d\theta \, d\phi$

(i)

$$\int_0^{2\pi} L^e (R, \theta; \rho, \phi) * \cos[n\theta] \, d\theta$$

$$= \int_0^{2\pi} \left(\frac{1}{\rho} + \sum_{m=1}^{\infty} \frac{R^m}{\rho^{m+1}} \cos[m(\theta - \phi)] \right) * \cos[n\theta] \, d\theta$$

$=$

$$\int_0^{2\pi}\left\{\frac{1}{\rho}*\text{Cos}[n\,\theta]+\left(\sum_{m=1}^{\infty}\frac{R^m}{\rho^{m+1}}\,(\text{Cos}[m\,\theta]\,\text{Cos}[m\,\phi]+\text{Sin}[m\,\theta]\,\text{Sin}[m\,\phi])*\text{Cos}[n\,\theta]\right)\right\}d\theta$$

$$=\int_0^{2\pi}\left\{\frac{1}{\rho}*\text{Cos}[n\,\theta]+\frac{R^n}{\rho^{n+1}}\,\text{Cos}^2[n\,\theta]\,\text{Cos}[n\,\phi]\right\}d\theta$$

$$=\frac{1}{\rho}*\left(\frac{1}{n}\right)*\text{Sin}[n\,\theta]\,\Big|_0^{2\pi}+\frac{R^n}{\rho^{n+1}}\left(\frac{\theta}{2}+\frac{\text{Sin}[2\,n\,\theta]}{4\,n}\right)\text{Cos}[n\phi]\,\Big|_0^{2\pi}$$

$$=\frac{R^n}{\rho^{n+1}}\,\pi\,\text{Cos}[n\,\phi]$$

(ii)

$$\int_0^{2\pi}L^i\left(\rho,\,\phi;\,\bar{\rho},\,\bar{\phi}\right)\int_0^{2\pi}L^e\,(R,\,\theta;\,\rho,\,\phi)*\text{Cos}[n\,\theta]\,d\theta\,d\phi$$

$$=\int_0^{2\pi}\left(-\sum_{m=1}^{\infty}\frac{(\bar{\rho})^{m-1}}{\rho^m}\,\text{Cos}\left[m\,\left(\phi-\bar{\phi}\right)\right]\right)\left(\frac{R^n}{\rho^{n+1}}\,\pi\,\text{Cos}[n\,\phi]\right)d\phi$$

$$=\int_0^{2\pi}\left(-\sum_{m=1}^{\infty}\frac{(\bar{\rho})^{m-1}}{\rho^m}\,\left(\text{Cos}[m\,\phi]\,\text{Cos}\left[m\,\bar{\phi}\right]+\text{Sin}[m\,\phi]\,\text{Sin}\left[m\,\bar{\phi}\right]\right)\right)\left(\frac{R^n}{\rho^{n+1}}\,\pi\,\text{Cos}[n\,\phi]\right)d\phi$$

$$=\int_0^{2\pi}-\frac{(\bar{\rho})^{n-1}}{\rho^n}\;\frac{R^n}{\rho^{n+1}}\,\pi\,\text{Cos}^2[n\,\phi]\,\text{Cos}\left[n\,\bar{\phi}\right]d\phi$$

$$=-\frac{(\bar{\rho})^{n-1}\,R^n}{\rho^{2\,n+1}}\,\pi\left(\frac{\phi}{2}+\frac{\text{Sin}[2\,n\,\phi]}{4\,n}\right)\text{Cos}\left[n\,\bar{\phi}\right]\,\Big|_0^{2\pi}$$

$$=-\frac{(\bar{\rho})^{n-1}\,R^n}{\rho^{2\,n+1}}\,\pi^2\,\text{Cos}\left[n\,\bar{\phi}\right]$$

$\Longrightarrow(\rho=\bar{\rho}=R=1)$

$$=\boxed{-\pi^2\,\text{Cos}\left[n\,\bar{\phi}\right]}$$

● $\int_0^{2\pi}U\left(\rho,\,\phi;\,\bar{\rho},\,\bar{\phi}\right)\int_0^{2\pi}M\,(R,\,\theta;\,\rho,\,\phi)*\text{Cos}[n\theta]\,d\theta\,d\phi$

(i)

$$\int_0^{2\pi}M\,(R,\,\theta;\,\rho,\,\phi)\,d\theta$$

$$=\int_0^{2\pi}\left(\sum_{m=1}^{\infty}\frac{R^{m-1}}{\rho^{m+1}}\,m\,\text{Cos}[m\,(\theta-\phi)]\right)*\text{Cos}[n\,\theta]\,d\theta$$

$$=\int_0^{2\pi}\left(\sum_{m=1}^{\infty}\frac{R^{m-1}}{\rho^{m+1}}\,m\,(\text{Cos}[m\,\theta]\,\text{Cos}[m\,\phi]+\text{Sin}[m\,\theta]\,\text{Sin}[m\,\phi])\right)*\text{Cos}[n\,\theta]\,d\theta$$

$$=\int_0^{2\pi}\frac{R^{n-1}}{\rho^{n+1}}\,n\,\text{Cos}^2[n\,\theta]\,\text{Cos}[n\,\phi]\,d\theta$$

$$=\frac{R^{n-1}}{\rho^{n+1}}\,n\left(\frac{\theta}{2}+\frac{\text{Sin}[2\,n\,\theta]}{4\,n}\right)\text{Cos}[n\,\phi]\,\Big|_0^{2\pi}$$

$$=\frac{R^{n-1}}{\rho^{n+1}}\,n\,\pi\,\text{Cos}[n\,\phi]$$

(ii)

$$\int_0^{2\pi}U\left(\rho,\,\phi;\,\bar{\rho},\,\bar{\phi}\right)\int_0^{2\pi}M\,(R,\,\theta;\,\rho,\,\phi)*\text{Cos}[n\,\theta]\,d\theta\,d\phi$$

$$= \int_0^{2\pi} \left(\ln[\rho] - \sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{\bar{\rho}}{\rho}\right)^m \mathrm{Cos}\left[m\left(\phi - \bar{\phi}\right)\right] \right) * \left(\frac{R^{n-1}}{\rho^{n+1}} n \pi \mathrm{Cos}[n\phi]\right) d\phi$$

$$= \int_0^{2\pi} \left(\ln[\rho] * \left(\frac{R^{n-1}}{\rho^{n+1}} n \pi \mathrm{Cos}[n\phi]\right) - \sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{\bar{\rho}}{\rho}\right)^m \mathrm{Cos}\left[m\left(\phi - \bar{\phi}\right)\right] * \left(\frac{R^{n-1}}{\rho^{n+1}} n \pi \mathrm{Cos}[n\phi]\right) \right) d\phi$$

$$= \ln[\rho] \frac{R^{n-1}}{\rho^{n+1}} \pi n \left(\frac{1}{n}\right) \mathrm{Sin}[n\phi] \Big|_0^{2\pi} - \int_0^{2\pi} \frac{(\bar{\rho})^n R^{n-1}}{\rho^{2n+1}} \pi \mathrm{Cos}^2[n\phi] \mathrm{Cos}[n\bar{\phi}] d\phi$$

$$= -\frac{(\bar{\rho})^n R^{n-1}}{\rho^{2n+1}} \pi \left(\frac{\theta}{2} + \frac{\mathrm{Sin}[2n\theta]}{4n}\right) \mathrm{Cos}[n\bar{\phi}] \Big|_0^{2\pi}$$

$$= -\frac{(\bar{\rho})^n R^{n-1}}{\rho^{2n+1}} \pi^2 \mathrm{Cos}[n\bar{\phi}]$$

$\Longrightarrow (\rho = \bar{\rho} = R = 1)$

$$= \boxed{-\pi^2 \mathrm{Cos}[n\bar{\phi}]}$$

- $\int_0^{2\pi} M\left(\rho, \phi; \bar{\rho}, \bar{\phi}\right) \int_0^{2\pi} U(R, \theta; \rho, \phi) * \mathrm{Cos}[n\theta] \, d\theta \, d\phi$

(i)

$$\int_0^{2\pi} U(R, \theta; \rho, \phi) \, d\theta$$

$$= \int_0^{2\pi} \left(\ln[\rho] - \sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{R}{\rho}\right)^m \mathrm{Cos}[m(\theta - \phi)] \right) * \mathrm{Cos}[n\theta] \, d\theta$$

$$= \int_0^{2\pi} \left(\ln[\rho] * \mathrm{Cos}[n\theta] - \left(\sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{R}{\rho}\right)^m \mathrm{Cos}[m(\theta - \phi)] \right) * \mathrm{Cos}[n\theta] \right) d\theta$$

$$= \int_0^{2\pi} \left(\ln[\rho] * \mathrm{Cos}[n\theta] - \left(\sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{R}{\rho}\right)^m (\mathrm{Cos}[m\theta] \mathrm{Cos}[m\phi] + \mathrm{Sin}[m\theta] \mathrm{Sin}[m\phi]) \right) * \mathrm{Cos}[n\theta] \right) d\theta$$

$$= \int_0^{2\pi} \left(\ln[\rho] * \mathrm{Cos}[n\theta] - \left(\frac{1}{n} \left(\frac{R}{\rho}\right)^n \mathrm{Cos}^2[n\theta] \mathrm{Cos}[n\phi] \right) \right) d\theta$$

$$= \ln[\rho] * \frac{1}{n} * \mathrm{Sin}[n\theta] \Big|_0^{2\pi} - \frac{1}{n} \left(\frac{R}{\rho}\right)^n \left(\frac{\theta}{2} + \frac{\mathrm{Sin}[2n\theta]}{4n}\right) \mathrm{Cos}[n\phi] \Big|_0^{2\pi}$$

$$= -\frac{1}{n} \left(\frac{R}{\rho}\right)^n \pi \mathrm{Cos}[n\phi]$$

(ii)

$$\int_0^{2\pi} M\left(\rho, \phi; \bar{\rho}, \bar{\phi}\right) \int_0^{2\pi} U(R, \theta; \rho, \phi) * \mathrm{Cos}[n\theta] \, d\theta \, d\phi$$

$$= \int_0^{2\pi} \left(\sum_{m=1}^{\infty} \frac{(\bar{\rho})^{m-1}}{\rho^{m+1}} m \mathrm{Cos}\left[m\left(\phi - \bar{\phi}\right)\right] \left(-\frac{1}{n} \left(\frac{R}{\rho}\right)^n \pi \mathrm{Cos}[n\phi] \right) \right) d\phi$$

$$= \int_0^{2\pi} \left(\frac{(\bar{\rho})^{n-1}}{\rho^{n+1}} n \mathrm{Cos}^2[n\phi] \mathrm{Cos}[n\bar{\phi}] \left(\frac{-1}{n}\right) \left(\frac{R}{\rho}\right)^n \pi \right) d\phi$$

$$= -\frac{(\bar{\rho})^{n-1} R^n}{\rho^{2n+1}} \left(\frac{\theta}{2} + \frac{\mathrm{Sin}[2n\theta]}{4n}\right) \mathrm{Cos}[n\bar{\phi}] \pi \Big|_0^{2\pi}$$

$$= -\frac{(\bar{\rho})^{n-1} R^n}{\rho^{2n+1}} \pi^2 \mathrm{Cos}[n\bar{\phi}]$$

$\Longrightarrow (\rho = \bar{\rho} = R = 1)$

$$= \boxed{-\pi^2 \mathrm{Cos}[n\bar{\phi}]}$$