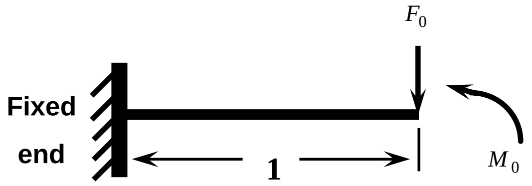
Indirect method for 1-D beam problem



$$u(0)=0,\theta(0)=0$$
$$m(1)=1,v(1)=1$$

$$\frac{d^4u(x)}{dx^4}=0,\ \ 0<x<1$$

$$\boxed{u(x)=\frac{x^3}{6},\ \ 0<x<1}$$

$$U(s,x)=\delta(s-x)$$

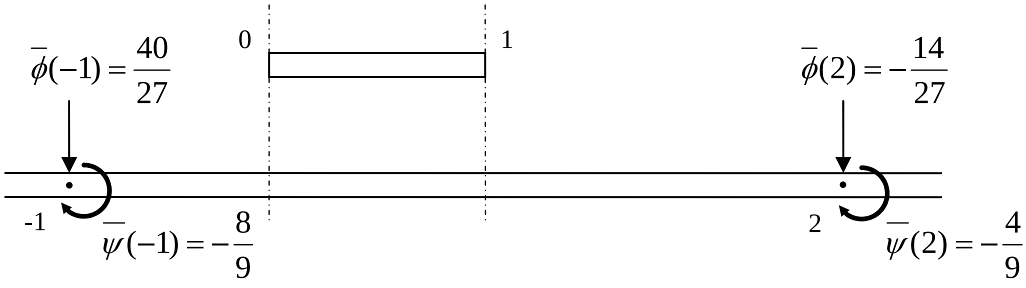
$$u(x)=U(s,x)\phi(s)\,|_{s=-1}^{s=2}+\Theta(s,x)\psi(s)\,|_{s=-1}^{s=2}$$
$$=U(2,x)\phi(2)-U(-1,x)\phi(-1)+\Theta(2,x)\psi(2)-\Theta(-1,x)\psi(-1)$$
$$=\frac{(2-x)^3}{12}\phi(2)-\frac{(x+1)^3}{12}\phi(-1)-\frac{(2-x)^2}{4}\psi(2)-\frac{(x+1)^2}{4}\psi(-1)$$
$$=\frac{x^3}{6}$$

$$U(2,x)=\frac{(2-x)^3}{12},U(-1,x)=\frac{(x+1)^3}{12},\Theta(2,x)=-\frac{(2-x)^2}{4},\Theta(-1,x)=\frac{(x+1)^2}{4}$$

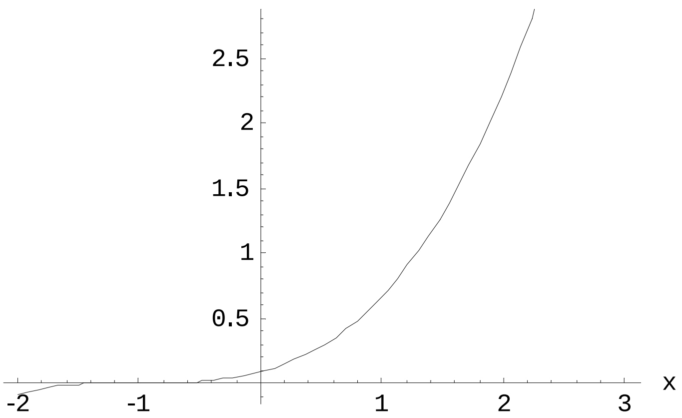
$$\phi(2)=-\frac{14}{27},\phi(-1)=-\frac{40}{27},\psi(2)=-\frac{4}{9},\psi(-1)=\frac{8}{9}$$

Indirect method	
Mathematical	**Physical**
$u(x)=U(s,x)\phi(s)\,\|_{s=-1}^{s=2}+\Theta(s,x)\psi(s)\,\|_{s=-1}^{s=2}$	$u(x)=\sum_{j=1}^{2}U(s_j,x)\bar{\phi}_j+\sum_{j=1}^{2}\Theta(s_j,x)\bar{\psi}_j$
$\phi(2)=-\frac{14}{27},\phi(-1)=-\frac{40}{27}$ $\psi(2)=-\frac{4}{9},\psi(-1)=\frac{8}{9}$	$\bar{\phi}(2)=-\frac{14}{27}=\phi(2),\bar{\phi}(-1)=\frac{40}{27}=-\phi(-1)$ $\bar{\psi}(2)=-\frac{4}{9}=\psi(2),\bar{\psi}(-1)=-\frac{8}{9}=-\psi(-1)$

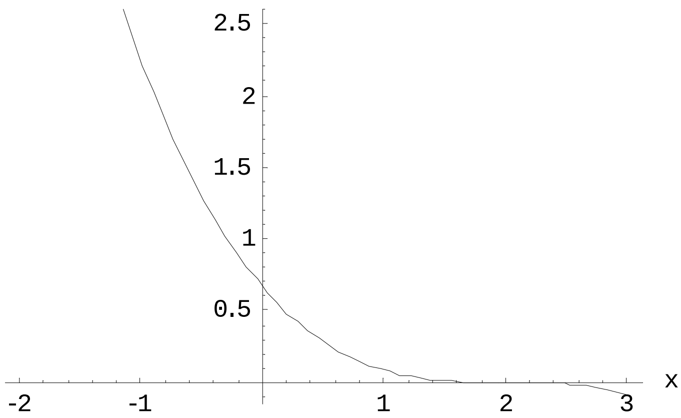
Solved problem



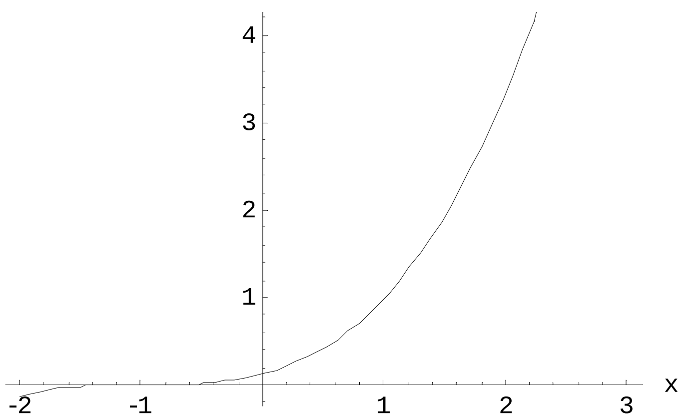
Imbedded to infinite beam



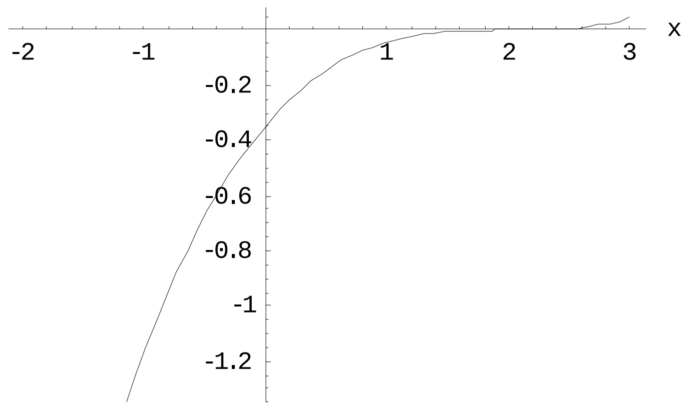
$U(-1,x)$



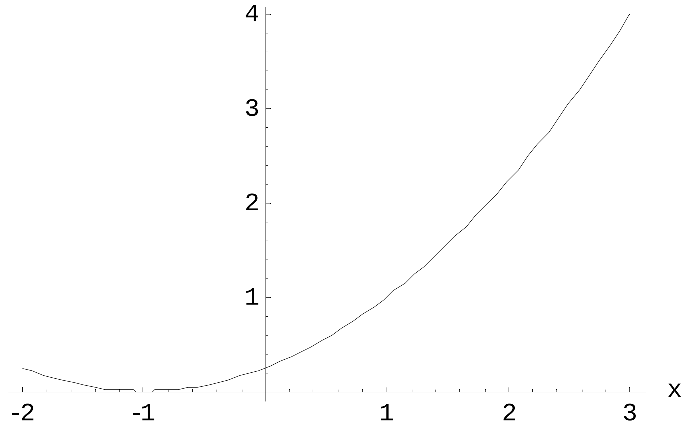
$U(2,x)$



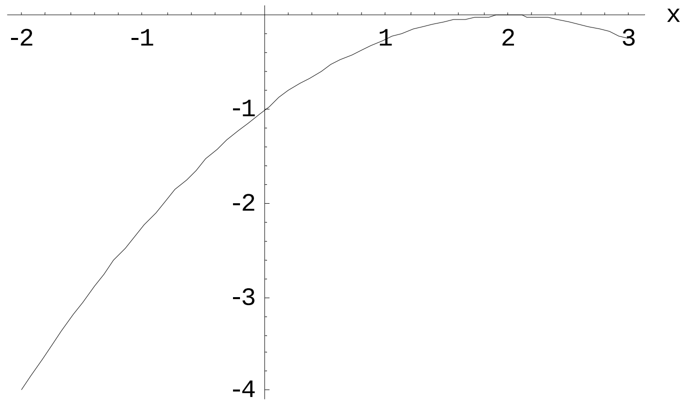
$U(-1,x)\bar{\phi}(-1) = -U(-1,x)\phi(-1)$



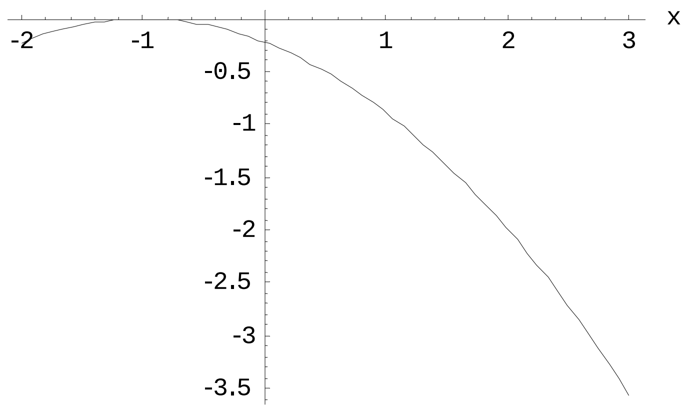
$U(2,x)\bar{\phi}(2) = U(2,x)\phi(2)$



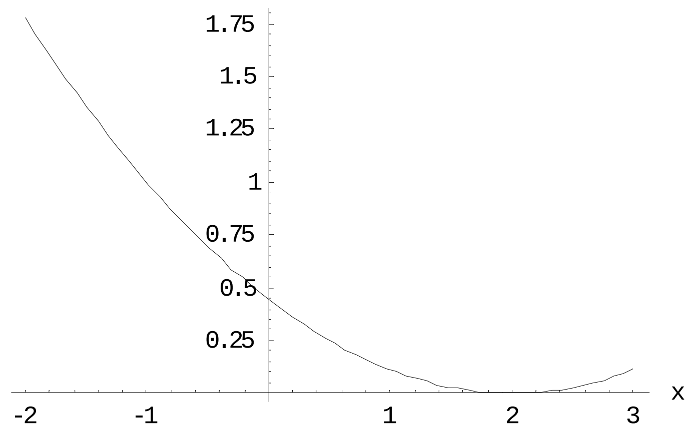
$\Theta(-1, x)$



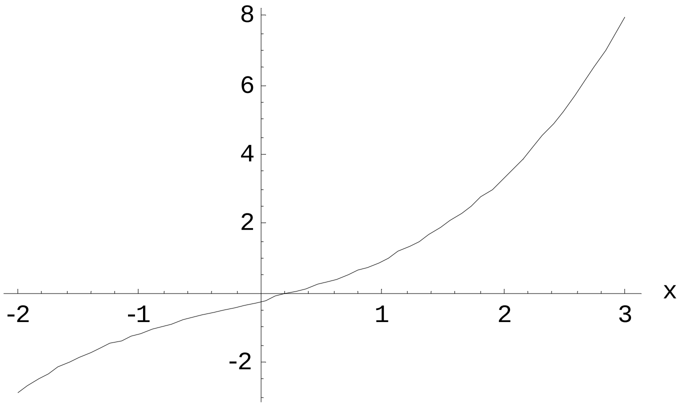
$\Theta(2, x)$



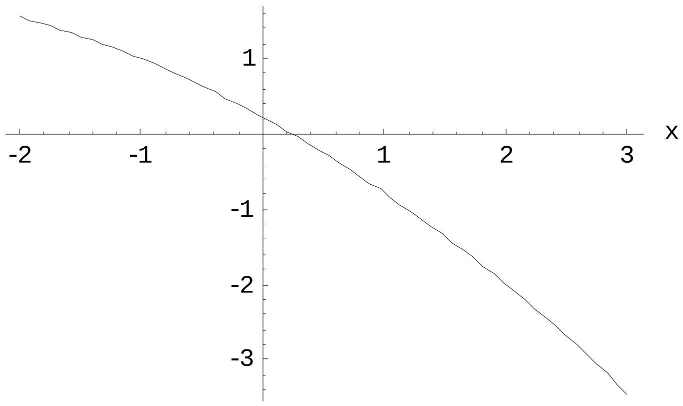
$\Theta(-1, x)\overline{\psi}(-1) = -\Theta(-1, x)\psi(-1)$



$\Theta(2, x)\overline{\psi}(2) = \Theta(2, x)\psi(2)$

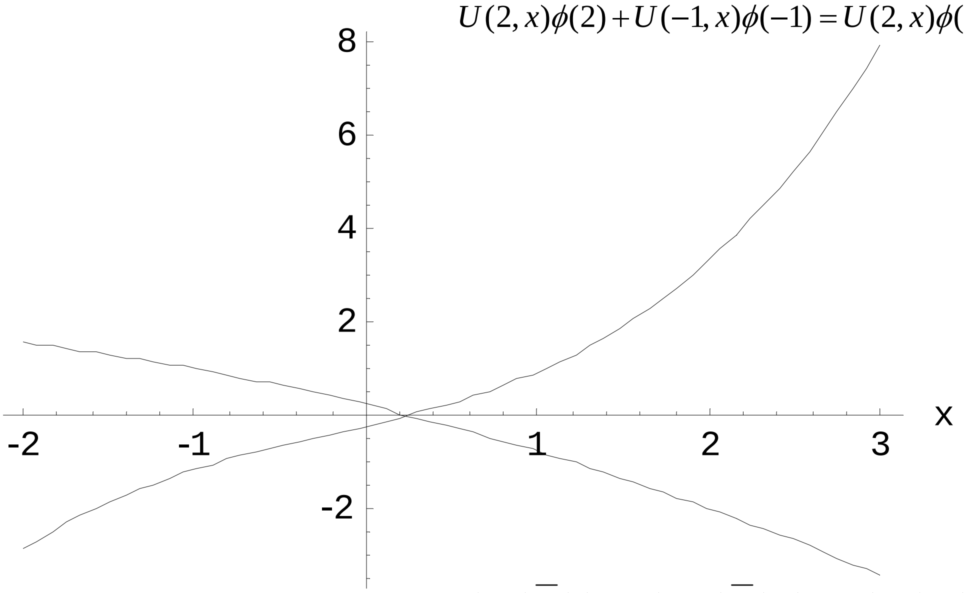


$$U(2,x)\overline{\phi}(2)+U(-1,x)\overline{\phi}(-1)=U(2,x)\phi(2)-U(-1,x)\phi(-1)$$

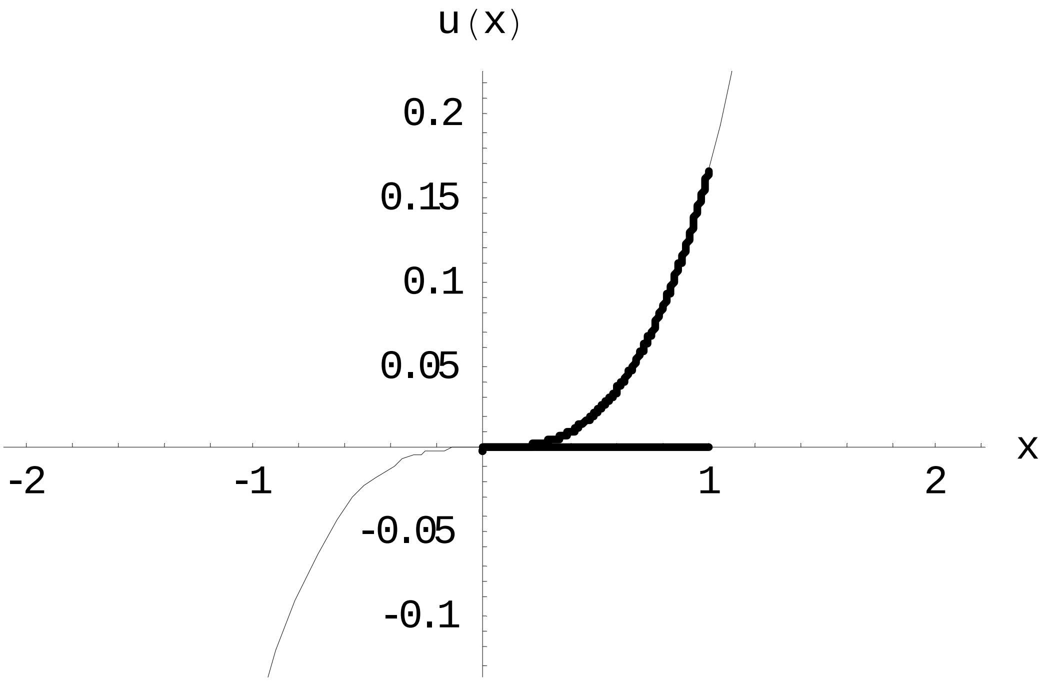


$$\Theta(2,x)\overline{\psi}(2)+\Theta(-1,x)\overline{\psi}(-1)=\Theta(2,x)\psi(2)-\Theta(-1,x)\psi(-1)$$

$$U(2,x)\overline{\phi}(2)+U(-1,x)\overline{\phi}(-1)=U(2,x)\phi(2)-U(-1,x)\phi(-1)$$



$$\Theta(2,x)\overline{\psi}(2)+\Theta(-1,x)\overline{\psi}(-1)=\Theta(2,x)\psi(2)-\Theta(-1,x)\psi(-1)$$



$$
\begin{aligned}
&U(2,x)\overline{\phi}(2)+U(-1,x)\overline{\phi}(-1)+\Theta(2,x)\overline{\psi}(2)+\Theta(-1,x)\overline{\psi}(-1)\\
&=U(2,x)\phi(2)-U(-1,x)\phi(-1)+\Theta(2,x)\psi(2)-\Theta(-1,x)\psi(-1)
\end{aligned}
$$