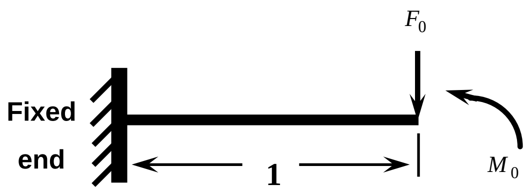
Indirect method for 1-D beam problem (U,M)



$$u(0)=0, v(0)=0$$
$$m(1)=2, v(1)=1$$

$$\frac{d^4u(x)}{dx^4}=0,\ 0<x<1$$

$$\boxed{u(x)=\frac{1}{2}x^2+\frac{1}{6}x^3, 0<x<1}$$

$$U(s,x)=\delta(s-x)$$

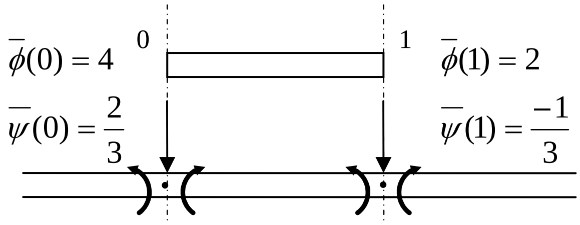
$$U(1,x)=\frac{1}{12}(1-x)^3, U(0,x)=\frac{1}{12}x^3, M(1,x)=\frac{1}{2}(1-x), M(0,x)=\frac{1}{2}x$$
$$\phi(0)=-4, \phi(1)=2, \psi(0)=\frac{-2}{3}, \psi(1)=\frac{-1}{3}$$

$$u(x)=U(s,x)\phi(s)+M(s,x)\varphi(s)$$

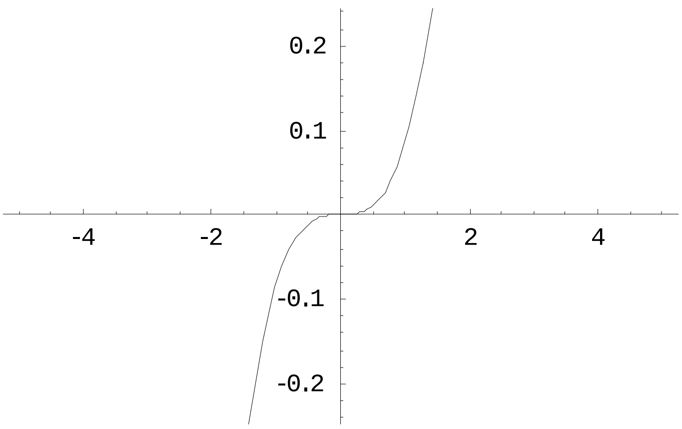
$$u(x)=U(1,x)\phi(1)+M(1,x)\varphi(1)-U(0,x)\phi(0)-M(0,x)\varphi(0)$$
$$=(\frac{1}{12}(1-x)^3(-2)(1-2))+(\frac{1}{2}(1-x)(\frac{1-2}{3}))-(\frac{1}{12}x^3(-2)(2))-((\frac{1}{2}x)\frac{2}{3}(1-2)$$
$$=\frac{1}{6}(3(2)+(-3+x))x^2$$
$$=\frac{1}{6}(3+x)x^2$$

Indirect method	
Mathematical	**Physical**
$u(x)=U(s,x)\phi(s)\Big\vert_{s=0}^{s=1}+M(s,x)\psi(s)\Big\vert_{s=0}^{s=1}$	$u(x)=\sum_{s=1}^{2}U(s,x)\overline{\phi}(s)+\sum_{s=1}^{2}M(s,x)\overline{\psi}(s)$
$\phi(0)=-4, \phi(1)=2$ $\psi(0)=\frac{-2}{3}, \psi(1)=\frac{-1}{3}$	$\overline{\phi}(0)=4=-\phi(0), \overline{\phi}(1)=2$ $\overline{\psi}(0)=\frac{2}{3}=-\psi(0), \overline{\psi}(1)=\frac{-1}{3}$

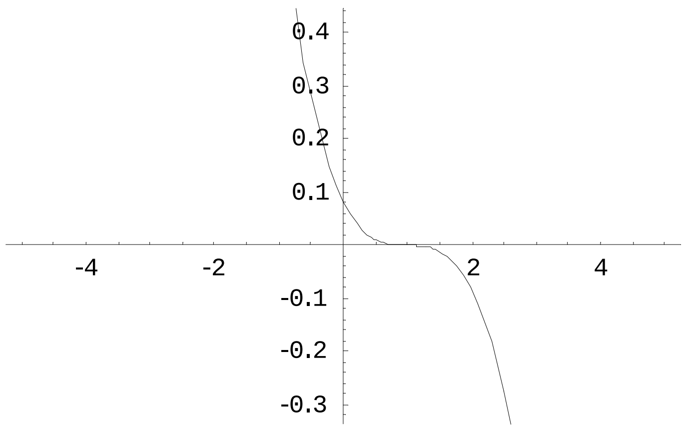
Solved problem



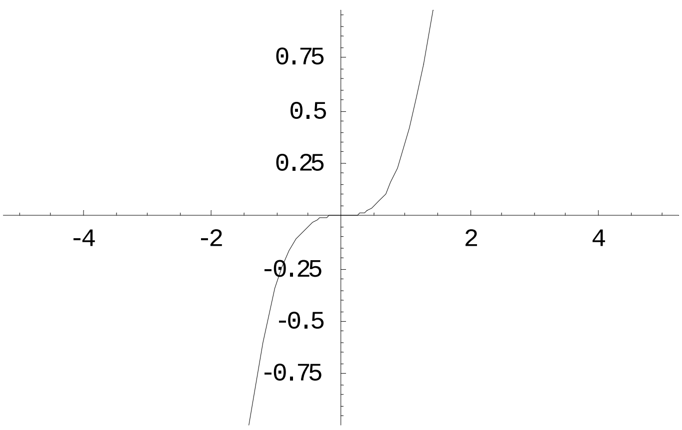
Imbedded to infinite beam



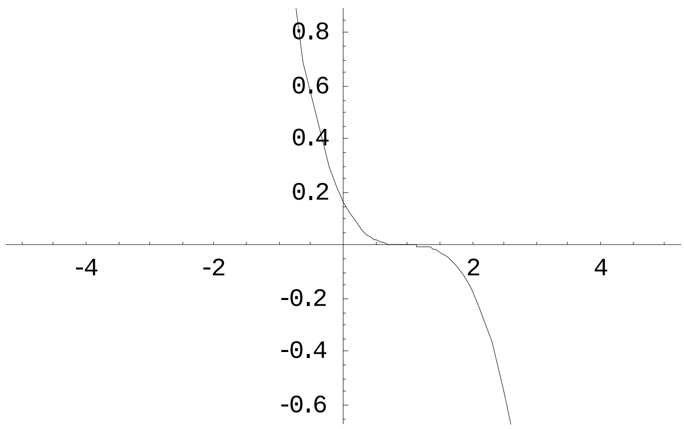
$U(0, x)$



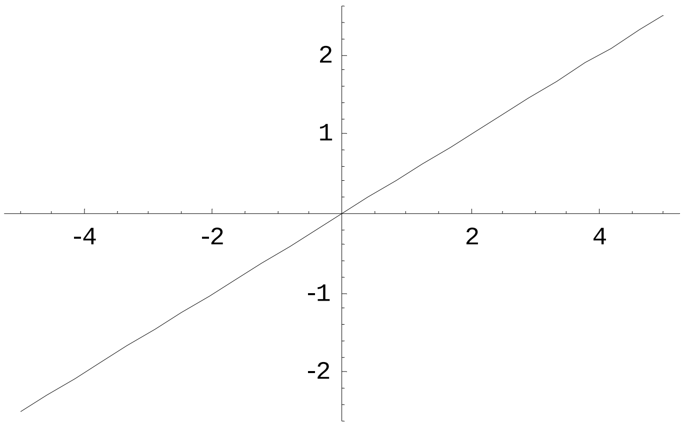
$U(-1, x)$



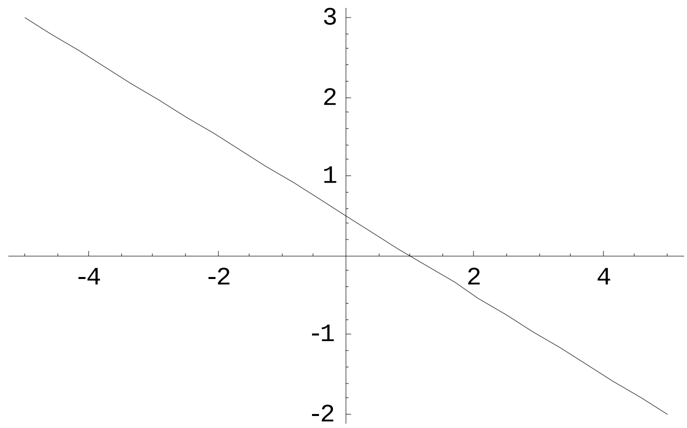
$U(0, x)\bar{\phi}(0)$



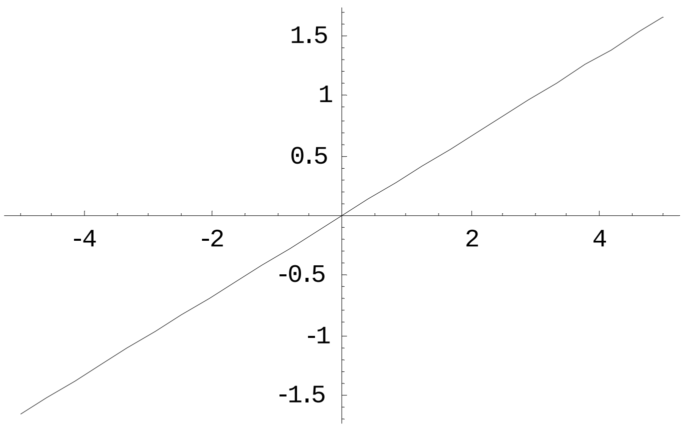
$U(1, x)\bar{\phi}(1)$



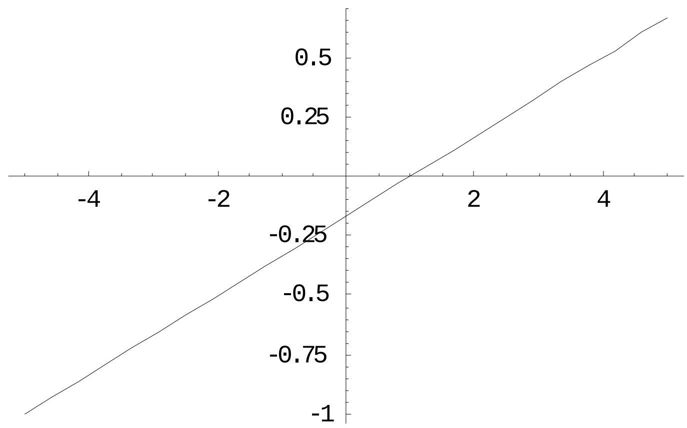
$M(0,x)$



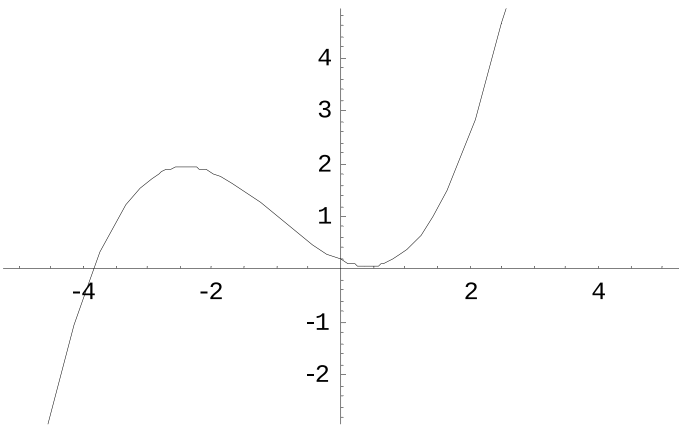
$M(1,x)$



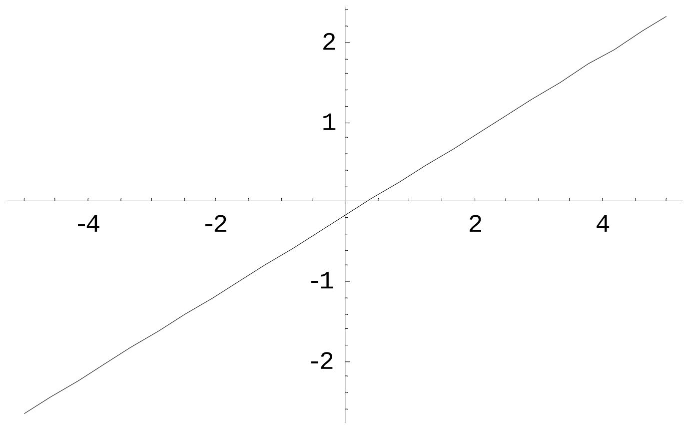
$M(0,x)\overline{\psi}(0)$



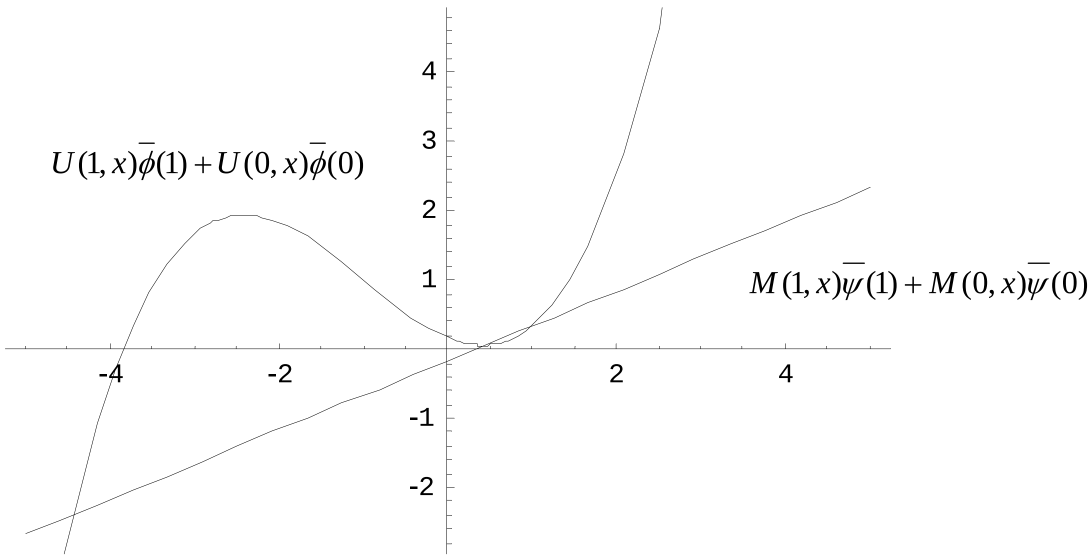
$M(1,x)\overline{\psi}(1)$

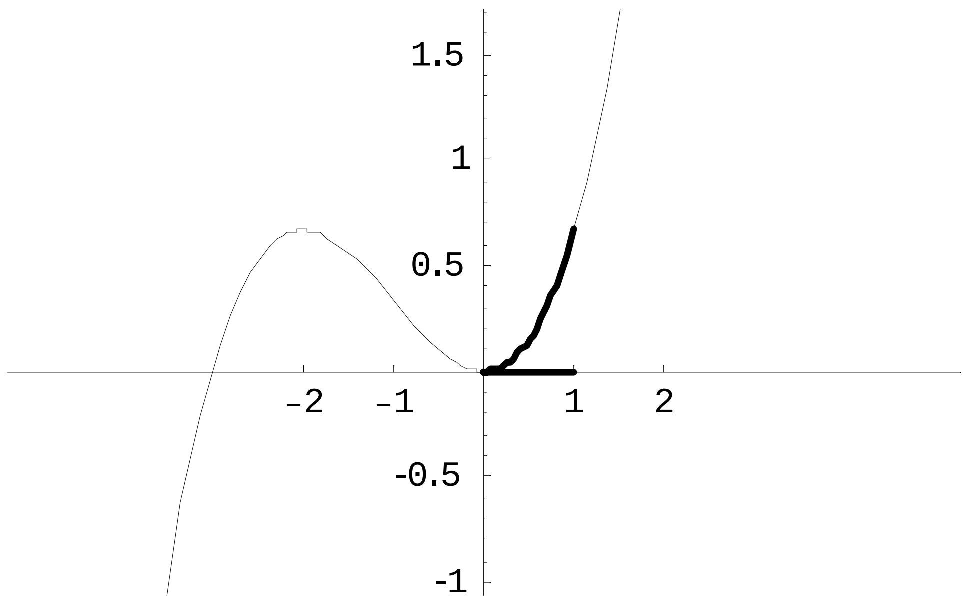


$U(1,x)\phi(1)-U(0,x)\phi(0)=U(1,x)\overline{\phi}(1)+U(0,x)\overline{\phi}(0)$



$M(1,x)\psi(1)-M(0,x)\psi(0)=M(1,x)\overline{\psi}(1)+M(0,x)\overline{\psi}(0)$





$$U(1,x)\phi(1)-U(0,x)\phi(0)+M(1,x)\psi(1)-M(0,x)\psi(0)$$
$$=U(1,x)\overline{\phi}(1)+U(0,x)\overline{\phi}(0)+M(1,x)\overline{\psi}(1)+M(0,x)\overline{\psi}(0)$$