平 面 曲 線 中 曲 率 半 徑 的 解 法　　作業1

Method 1：(Frenet formula)

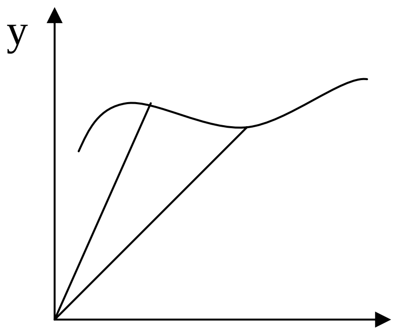
$$\underset{\sim}{X}(s) = (X(s), Y(s), 0)$$

$$\underset{\sim}{\dot{X}}(s) = (\dot{X}(s), \dot{Y}(s), 0)$$

$$\underset{\sim}{\tau}(s) = \frac{\underset{\sim}{\dot{X}}(s)}{\left\| \underset{\sim}{\dot{X}}(s) \right\|} = \left(\frac{\dot{X}(s)}{\sqrt{\dot{X}(s)^2 + \dot{Y}(s)^2}}, \frac{\dot{Y}(s)}{\sqrt{\dot{X}(s)^2 + \dot{Y}(s)^2}}, 0 \right)$$

$$\left\| \underset{\sim}{\dot{\tau}}(s) \right\| = \frac{1}{\rho} \quad \rightarrow \quad \rho = \frac{(\dot{X}^2 + \dot{Y}^2)}{(\dot{X}\ddot{Y} - \ddot{X}\dot{Y})}$$

Method 2：(微積分想法)



$$\tan\theta = y'(x)$$

$$\tan(\theta + \Delta\theta) = y'(x + \Delta x)$$

$$\tan(\theta + \Delta\theta) - \tan(\theta) = y'(x + \Delta x) - y'(x)$$

$$\sec^2\theta d\theta = y''(x)dx$$

$$(1 + (y')^2)d\theta = y'' \frac{ds}{\sqrt{1 + (y')^2}}$$

$$\frac{ds}{d\theta} = \frac{(1 + (y'(x))^2)^{3/2}}{y''}$$

$$\rho = \frac{(1 + (y')^2)^{3/2}}{\left\| y'' \right\|}$$

Link the results of Methods 1 and 2

Hint ： $y'(x) = \frac{dy(x)}{dx} = \frac{dY(s)}{dX(s)} = \frac{Y'(s)}{X'(s)}$