國立台灣海洋大學工程數學(三) 第三次作業

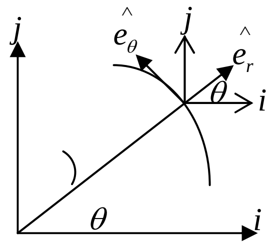
$x-y$ coordinate	$r-\theta$ coordinate
$u(x,y)$ Scalar function	$U(r,\theta)$ Scalar function
$\underset{\sim}{u}(x.y)=u_1(x,y)\underset{\sim}{i}+u_2(x,y)\underset{\sim}{j}$	$\underset{\sim}{U}(r,\theta)=U_r(r,\theta)\hat{e}_r+U_\theta(r,\theta)\hat{e}_\theta$
$\frac{\partial}{\partial x}(\ )=\cos\theta\frac{\partial}{\partial r}(\ )-\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}(\ )$ $\frac{\partial}{\partial y}(\ )=\sin\theta\frac{\partial}{\partial r}(\ )+\frac{\cos\theta}{r}\frac{\partial}{\partial\theta}(\ )$	
$\nabla u=\frac{\partial u}{\partial x}\underset{\sim}{i}+\frac{\partial u}{\partial y}\underset{\sim}{j}$	$\nabla U=\frac{\partial U}{\partial r}\hat{e}_r+\frac{1}{r}\frac{\partial U}{\partial\theta}\hat{e}_\theta$
$\nabla\cdot\underset{\sim}{u}=\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}$	$\nabla\cdot\underset{\sim}{U}=\frac{\partial U_r}{\partial r}+\frac{1}{r}U_r+\frac{1}{r}\frac{\partial U_\theta}{\partial\theta}$
$\nabla^2u=\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}$	$\nabla^2U=\frac{\partial^2U}{\partial r^2}+\frac{1}{r}\frac{\partial U}{\partial r}+\frac{1}{r^2}\frac{\partial^2U}{\partial\theta^2}$

$\underset{\sim}{i}$, $\underset{\sim}{j}$ (unit vector)	$\hat{e}_r$, $\hat{e}_\theta$ (unit vector)
$\begin{Bmatrix} i \\ j \end{Bmatrix}=\begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}\begin{Bmatrix}\hat{e}_r \\ \hat{e}_\theta\end{Bmatrix}$	
$u_1(x,y)\underset{\sim}{i}+u_2(x,y)\underset{\sim}{j}$	$U_r(r,\theta)\hat{e}_r+U_\theta(r,\theta)\hat{e}_\theta$
$\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}=\begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}\begin{Bmatrix}U_r \\ U_\theta\end{Bmatrix}$	

Please derive Laplacian in polar coordinate

odd numbers ： $\nabla^2u=\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}$ to $\nabla^2U=\frac{\partial^2U}{\partial r^2}+\frac{1}{r}\frac{\partial U}{\partial r}+\frac{1}{r^2}\frac{\partial^2U}{\partial\theta^2}$ by using $\frac{\partial^2}{\partial x^2}(\ )$ and $\frac{\partial^2}{\partial y^2}(\ )$

even numbers ：use $\nabla^2u=\nabla\cdot(\nabla u)$



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