國立台灣海洋大學工程數學(三) 第三次作業解答

第一種解法：

$$\nabla^2 u = \nabla \bullet (\nabla u) = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial x}(\cos\theta \frac{\partial U}{\partial \gamma} - \frac{\sin\theta}{\gamma}\frac{\partial U}{\partial \theta}) + \frac{\partial}{\partial y}(\sin\theta \frac{\partial U}{\partial \gamma} + \frac{\cos\theta}{\gamma}\frac{\partial U}{\partial \theta})$$

$$= \left[\cos\theta \frac{\partial}{\partial \gamma}(\cos\theta \frac{\partial U}{\partial \gamma} - \frac{\sin\theta}{\gamma}\frac{\partial U}{\partial \theta}) - \frac{\sin\theta}{\gamma}\frac{\partial}{\partial \theta}(\cos\theta \frac{\partial U}{\partial \gamma} - \frac{\sin\theta}{\gamma}\frac{\partial U}{\partial \theta})\right]$$

$$+ \left[\sin\theta \frac{\partial}{\partial \gamma}(\sin\theta \frac{\partial U}{\partial \gamma} + \frac{\cos\theta}{\gamma}\frac{\partial U}{\partial \theta}) + \frac{\cos\theta}{\gamma}\frac{\partial}{\partial \theta}(\sin\theta \frac{\partial U}{\partial \gamma} + \frac{\cos\theta}{\gamma}\frac{\partial U}{\partial \theta})\right]$$

$$= [\cos^2\theta \frac{\partial^2 U}{\partial \gamma^2} + \frac{\sin\theta\cos\theta}{\gamma^2}\frac{\partial U}{\partial \theta} - \frac{\sin\theta\cos\theta}{\gamma}\frac{\partial^2 U}{\partial \gamma \partial \theta} + \frac{\sin^2\theta}{\gamma}\frac{\partial U}{\partial \gamma} - \frac{\sin\theta\cos\theta}{\gamma}\frac{\partial^2 U}{\partial \gamma \partial \theta}$$

$$+ \frac{\sin\theta\cos\theta}{\gamma^2}\frac{\partial U}{\partial \theta} + \frac{\sin^2\theta}{\gamma^2}\frac{\partial^2 U}{\partial \theta^2}] + [\sin^2\theta \frac{\partial^2 U}{\partial \gamma^2} - \frac{\sin\theta\cos\theta}{\gamma^2}\frac{\partial U}{\partial \theta} + \frac{\sin\theta\cos\theta}{\gamma}\frac{\partial^2 U}{\partial \gamma \partial \theta}$$

$$+ \frac{\cos^2\theta}{\gamma}\frac{\partial U}{\partial \gamma} + \frac{\sin\theta\cos\theta}{\gamma}\frac{\partial^2 U}{\partial \gamma \partial \theta} - \frac{\sin\theta\cos\theta}{\gamma^2}\frac{\partial U}{\partial \theta} + \frac{\cos^2\theta}{\gamma^2}\frac{\partial^2 U}{\partial \theta^2}]$$

$$= (\cos^2\theta + \sin^2\theta)\frac{\partial^2 U}{\partial \gamma^2} + \left(\frac{\sin^2\theta + \cos^2\theta}{\gamma}\right)\frac{\partial U}{\partial \gamma} + \left(\frac{\sin^2\theta + \cos^2\theta}{\gamma^2}\right)\frac{\partial^2 U}{\partial \theta^2}$$

$$= \frac{\partial^2 U}{\partial \gamma^2} + \frac{1}{\gamma}\frac{\partial U}{\partial \gamma} + \frac{1}{\gamma^2}\frac{\partial^2 U}{\partial \theta^2} \text{.....答}$$

第二種解法：

$$\nabla^2 u = \nabla \bullet (\nabla u) = \nabla \bullet (\nabla U) = \nabla \bullet \left[\frac{\partial U}{\partial \gamma}\hat{e}_r + \frac{1}{\gamma}\frac{\partial U}{\partial \theta}\hat{e}_\theta\right]$$

$$= \frac{\partial}{\partial \gamma}\left(\frac{\partial U}{\partial \gamma}\right) + \frac{1}{\gamma}\left(\frac{\partial U}{\partial \gamma}\right) + \frac{1}{\gamma}\frac{\partial}{\partial \theta}\left(\frac{1}{\gamma}\frac{\partial U}{\partial \theta}\right)$$

$$= \frac{\partial^2 U}{\partial \gamma^2} + \frac{1}{\gamma}\frac{\partial U}{\partial \gamma} + \frac{1}{\gamma^2}\frac{\partial^2 U}{\partial \theta^2} \text{.....答}$$