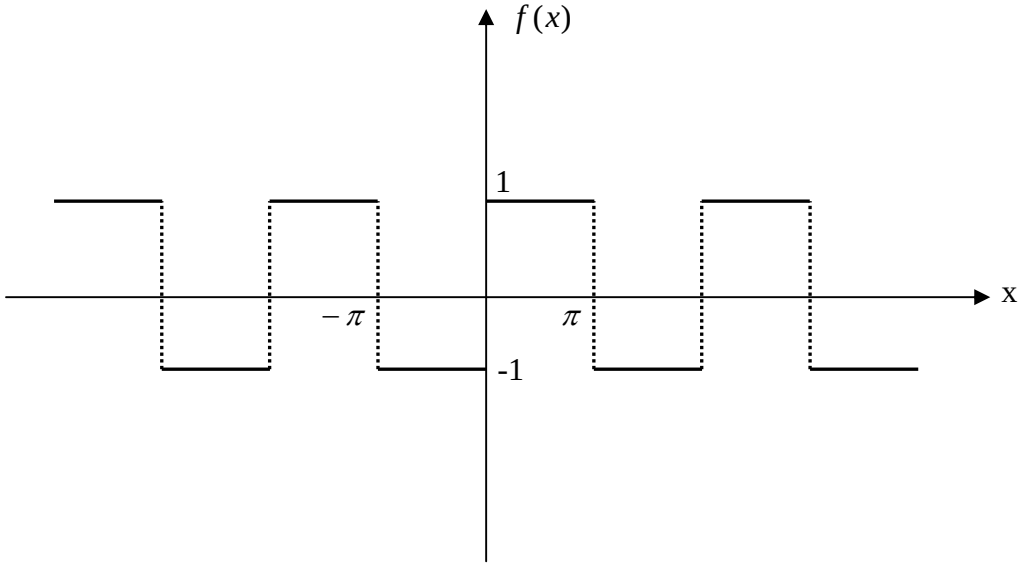
國立台灣海洋大學工程數學(三) 第四次作業解答



Expand the function into Fourier series and plot.

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$$

$f(x)$ 爲偶函數，所以 $a_0 = a_n = 0$

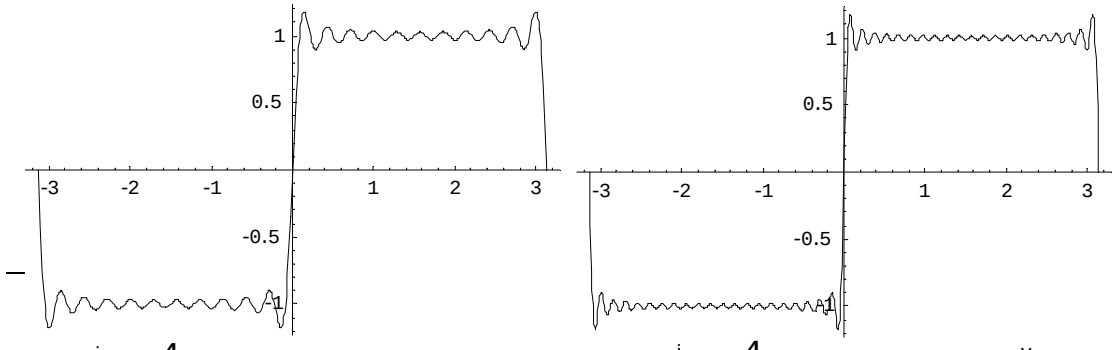
$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)dx = \frac{2}{\pi}\int_{0}^{\pi} \sin(nx)dx = \frac{-2}{n\pi}[\cos(n\pi)-1]$$

$$f(x) = \sum_{n=1}^{\infty} \frac{-2}{n\pi}[\cos(n\pi)-1]\sin(nx)$$

$$n = 2k+1, \quad b_n = \frac{-2}{n\pi}[\cos(n\pi)-1] = \frac{4}{(2k+1)\pi}$$

$$n = 2k, \qquad b_n = \frac{-2}{n\pi}[\cos(n\pi)-1] = 0$$

$$f(x) = \sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi}\sin[(2k+1)x]$$




```
f = Sum[4/(2 k + 1) π Sin[(2 k + 1) x], {k, 0, 10}]   f = Sum[4/(2 k + 1) π Sin[(2 k + 1) x], {k, 0, 20}]
```