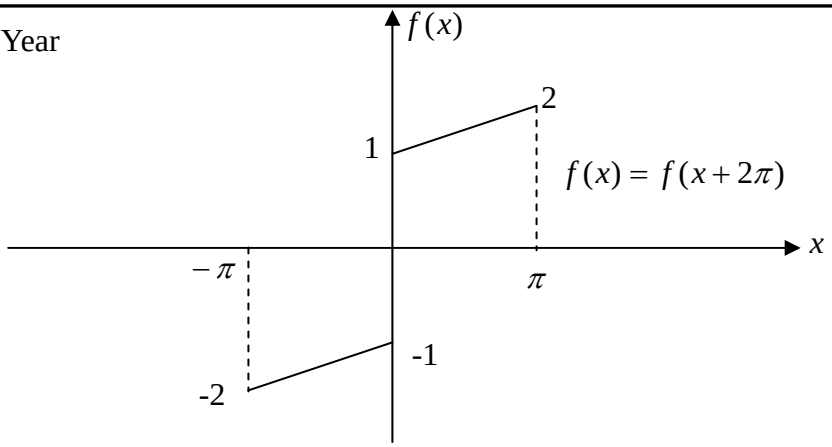
年級：＿＿＿＿＿＿　姓名：＿＿＿＿＿＿＿＿＿＿　學號：＿＿＿＿＿＿＿＿

國立台灣海洋大學河海工程學系２００４工程數學（三）第七次作業解答

1. Happy New Year



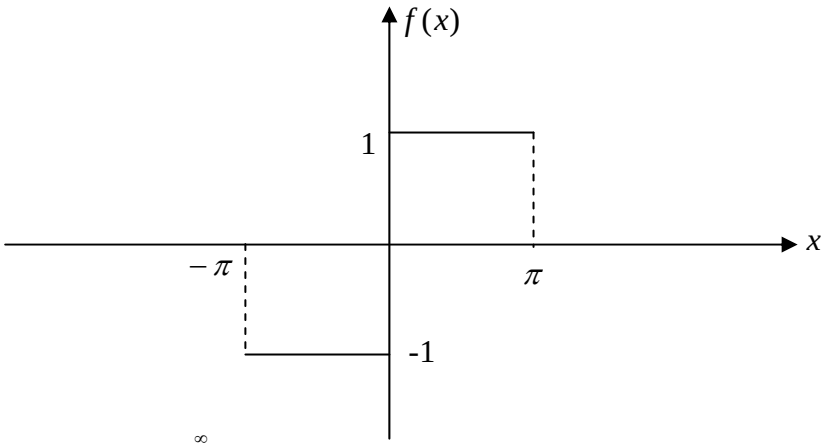
(1) Express $f(x)$ by using Fourier series.

(2) Express $f'(x)$ by using Fourier series.(termwise differentiation)

(3) Express $f'(x)$ by using Fourier series.(adding the lost terms)

(4) Express $f'(x)$ by using Fourier series.(Cesaro sum)

2.

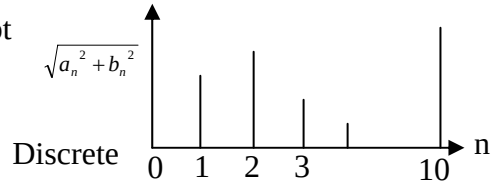


(1) Express $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$ where $f(x) = f(x+2\pi)$.

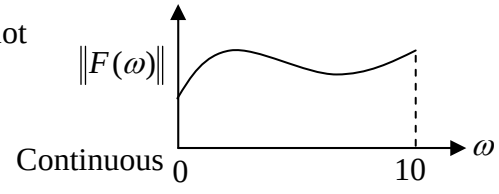
(2) Express $f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{i\omega x} d\omega$ where $f(x) = \begin{cases} 1, & 0 < x < \pi \\ -1, & -\pi < x < 0 \\ 0, & otherwise \end{cases}$ and

$F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx$.

Plot $\sqrt{a_n^2 + b_n^2}$



Plot $\|F(\omega)\|$



年級：_____　姓名：_____　學號：_____

國立台灣海洋大學河海工程學系２００４工程數學（三）第七次作業解答

1.

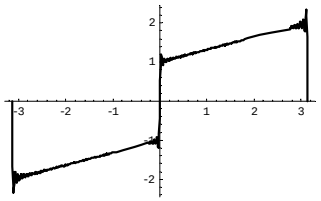
(1) $f(x)=\sum_{n=1}^{\infty}\frac{2}{n\pi}(1-2(-1)^n)\sin(nx)$

(2) $f'(x)=\sum_{n=1}^{\infty}\frac{2}{\pi}(1-2(-1)^n)\cos(nx)$

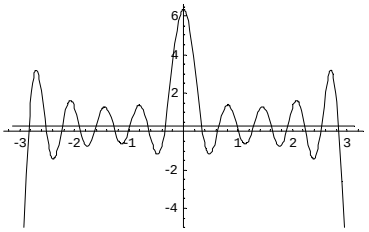
(3) $a_0=\frac{1}{2\pi}\int_{-\pi}^{\pi}f'(x)dx=\frac{1}{\pi}$,　$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f'(x)dx=0$,　$f'(x)=\frac{1}{\pi}$

(4) $f'(x)=\sum_{n=1}^{M}\frac{M-n+1}{M}[\frac{2}{\pi}-\frac{4}{\pi}(-1)^n]\cos(nx)$

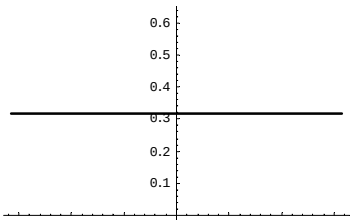
```
Plot[Sum[2/(n π) (1 - 2 (-1)^n) Sin[n x], {n, 1, 100}], {x, π, -π}]
```



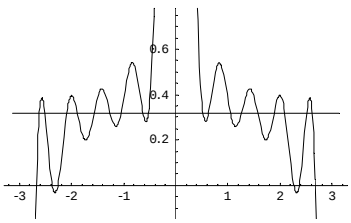
```
Plot[{Sum[2/π (1 - 2 (-1)^n) Cos[n x], {n, 1, 10}], 1/π}, {x, π, -π}]
```



```
Plot[1/π, {x, π, -π}]
```



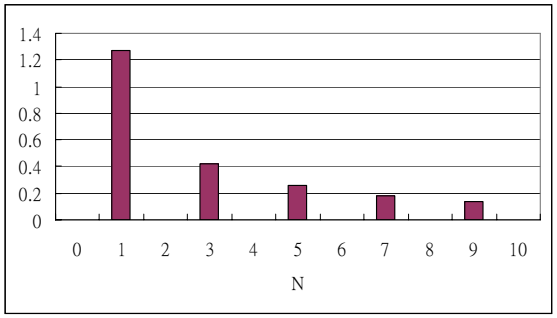
```
Plot[{Sum[(10 + 1 - n)/10 2/π (1 - 2 (-1)^n) Cos[n x], {n, 1, 10}], 1/π}, {x, π, -π}]
```



2.

(1) $a_0=a_n=0$,　$b_n=\frac{2}{n\pi}(1-(-1)^n)$,　$f(x)=\sum_{n=1}^{\infty}\frac{2}{n\pi}(1-(-1)^n)\sin(nx)$

(2) $F(\omega)=\int_{-\infty}^{\infty}f(x)e^{-i\omega x}dx=\int_{0}^{\pi}e^{-i\omega x}dx+\int_{-\pi}^{0}-e^{-i\omega x}dx=\frac{2}{i\omega}(1-\cos(\omega\pi))$




```
Plot[{Abs[2/w * (1 - (-1)^w)]}, {w, 0, 10}]
```