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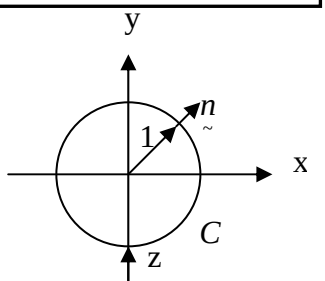
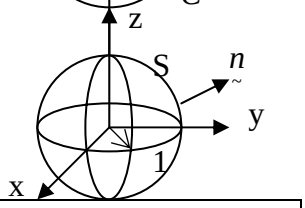
國立台灣海洋大學河海工程學系２００４工程數學（三）第一次大考模擬考解答

1. $\oint_C \underset{\sim}{r}\cdot\underset{\sim}{n}\,ds = ?$ where $\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j}$ (10%)



$$\oint_C \underset{\sim}{r}\cdot\underset{\sim}{n}\,ds = \iint_C \nabla\cdot\underset{\sim}{r}\,dA = 2A = 2\pi$$

2. $\iint_S \underset{\sim}{r}\cdot\underset{\sim}{n}\,dS = ?$ where $\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$ (10%)



$$\iint_S \underset{\sim}{r}\cdot\underset{\sim}{n}\,dS = \iiint \nabla\cdot\underset{\sim}{r}\,dV = \iiint 3dV = 3V = 3*\frac{4}{3}\pi r^3 = 4\pi$$

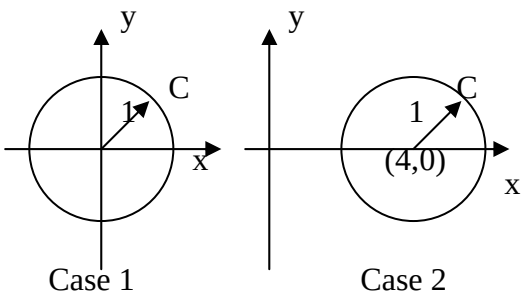
3. Fill in the Table (30%)

	1-D	2-D	3-D
r	$\sqrt{x^2}$	$\sqrt{x^2+y^2}$	$\sqrt{x^2+y^2+z^2}$
$\underset{\sim}{r}$	$x\underset{\sim}{i}$	$x\underset{\sim}{i}+y\underset{\sim}{j}$	$x\underset{\sim}{i}+y\underset{\sim}{j}+z\underset{\sim}{k}$
∇r	$\underset{\sim}{r}/r$	$\underset{\sim}{r}/r$	$\underset{\sim}{r}/r$
$\nabla\cdot\underset{\sim}{r}$	1	2	3
$\nabla\cdot(\nabla\times\underset{\sim}{r})$	0	0	0
$\nabla\cdot\nabla(\frac{1}{r})$	$\frac{2}{\lvert x\rvert^3}$	$\frac{1}{r^3}$	0
$\nabla\times(\nabla r)$	$\underset{\sim}{0}$	$\underset{\sim}{0}$	$\underset{\sim}{0}$

4. Explain why Green's theorem can be special case of Guass theorem and Stokes' theorem.(20%)

Green's $\oint P dx + Q dy = \iint (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}) dA$, Guass $\oint \underset{\sim}{v}\cdot\underset{\sim}{n}\,ds = \iint \nabla\cdot\underset{\sim}{v}\,dxdy$ $\underset{\sim}{v} = (Q, -P)$,

Stokes' $\oint \underset{\sim}{v}\cdot\underset{\sim}{t}\,ds = \iint \nabla\times\underset{\sim}{v}\cdot d\underset{\sim}{A}$ $\underset{\sim}{v} = (P, Q)$



5. $\oint_C \frac{x}{x^2+y^2}dx + \frac{y}{x^2+y^2}dy = ?$ (30%)

Case 1 $x = \cos\theta$ $y = \sin\theta$

$$\oint_C \frac{x}{x^2+y^2}dx + \frac{y}{x^2+y^2}dy = \oint \cos\theta(\sin\theta - \sin\theta)d\theta = 0$$

Case 2 $x = \cos\theta + 4$ $y = \sin\theta$

$$\oint_C \frac{x}{x^2+y^2}dx + \frac{y}{x^2+y^2}dy = \oint \frac{-4\sin\theta}{17+8\cos\theta}d\theta = 0$$