年級：＿＿＿＿＿＿　姓名：＿＿＿＿＿＿＿＿＿＿　學號：＿＿＿＿＿＿＿＿

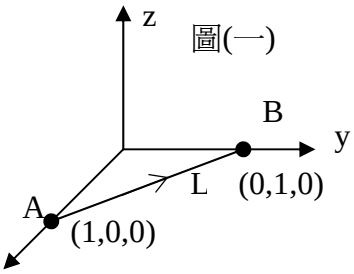
國立台灣海洋大學河海工程學系２００４工程數學（三）第一次大考(Nov. 19. 2004)

1.(1) $\nabla \cdot \underset{\sim}{r} = ?$ where $\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$.(5%)

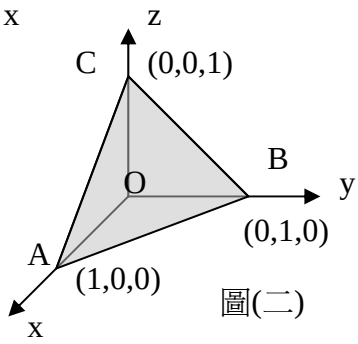
(2) $\oint_C \underset{\sim}{r} \cdot \underset{\sim}{n}\, ds = ?$ where L is the line of AB.(圖一)(10%)

(3) $\iint_{S_1} \underset{\sim}{r} \cdot \underset{\sim}{n}\, dS = ?$ where S_1 is the plane of OAC.(圖二)(5%)

(4) $\iint_{S_2} \underset{\sim}{r} \cdot \underset{\sim}{n}\, dS = ?$ where S_2 is the plane of ABC.(圖二)(5%)



圖(一)



圖(二)

2.(1) $\nabla \cdot (\nabla r) = ?$ (1-D)(5%)

(2) $\nabla \cdot (\nabla \ln r) = ?$ (2-D)(5%)

(3) $\nabla \cdot (\nabla \frac{1}{r}) = ?$ (3-D)(5%)

where r is the distance between $\underset{\sim}{x}$ and the origin.

3. (a) Find the radius of curvature at $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ for $y = \sqrt{1-x^2}$.(5%)

(b) Find the radius of curvature at (5.0) for $x = 4 + \cos t$, $y = \sin t$.(5%)

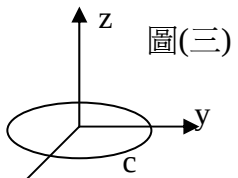
4.Give the vector field $\underset{\sim}{v} = y\underset{\sim}{i} - x\underset{\sim}{j} + z\underset{\sim}{k}$.

(1) Find $\nabla \times \underset{\sim}{v} = ?$ (5%)

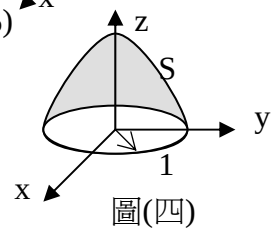
(2) Find the line integral $\oint_c \underset{\sim}{v} \cdot \underset{\sim}{t}\, ds = ?$ where c is $x^2 + y^2 = 1$.(圖三)(5%)

(3) Find the surface integral $\iint \nabla \times \underset{\sim}{v} \cdot dS = ?$ (5%)

where S is the hemispherical surface $x^2 + y^2 + z^2 = 1$.(圖四)



圖(三)



圖(四)

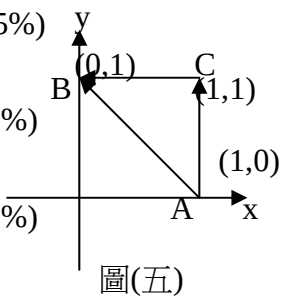
5.Explain why Green's theorem can be special case of Guass theorem and Stokes' theorem.(10%)

6.如圖五

(1) $\int_{AC} \frac{-y}{x^2+y^2} dx + \frac{x}{x^2+y^2} dy = ?$ (5%)　(4) $\int_{AC} \frac{x}{x^2+y^2} dx + \frac{y}{x^2+y^2} dy = ?$ (5%)

(2) $\int_{CB} \frac{-y}{x^2+y^2} dx + \frac{x}{x^2+y^2} dy = ?$ (5%)　(5) $\int_{CB} \frac{x}{x^2+y^2} dx + \frac{y}{x^2+y^2} dy = ?$ (5%)

(3) $\int_{AB} \frac{-y}{x^2+y^2} dx + \frac{x}{x^2+y^2} dy = ?$ (5%)　(6) $\int_{AB} \frac{x}{x^2+y^2} dx + \frac{y}{x^2+y^2} dy = ?$ (5%)



圖(五)