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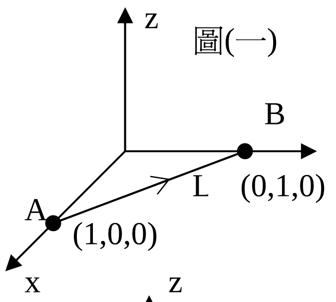
國立台灣海洋大學河海工程學系２００４工程數學（三）第一次大考解答(Nov. 19. 2004)

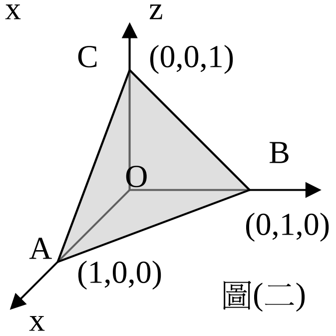
１.(1) $\nabla\cdot \underset{\sim}{r} = ?$ where $\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$.(5%)



(2) $\oint_C \underset{\sim}{r}\cdot\underset{\sim}{n}\, ds = ?$ where L is the line of AB.(圖一)(10%)

(3) $\iint_{S_1} \underset{\sim}{r}\cdot\underset{\sim}{n}\, dS = ?$ where S_1 is the plane of OAC.(圖二)(5%)

(4) $\iint_{S_2} \underset{\sim}{r}\cdot\underset{\sim}{n}\, dS = ?$ where S_2 is the plane of ABC.(圖二)(5%)



２.(1) $\nabla\cdot(\nabla r) = ?$ (1-D)(5%)

(2) $\nabla\cdot(\nabla \ln r) = ?$ (2-D)(5%)

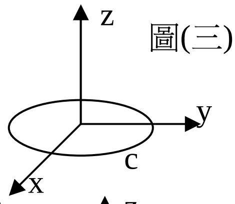
(3) $\nabla\cdot(\nabla \frac{1}{r}) = ?$ (3-D)(5%)

where r is the distance between $\underset{\sim}{x}$ and the origin.

３. (a) Find the radius of curvature at $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ for $y = \sqrt{1-x^2}$.(5%)

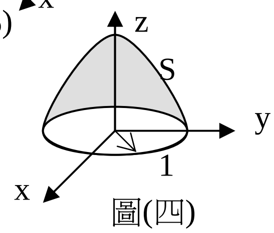
(b) Find the radius of curvature at (5.0) for $x = 4 + \cos t$, $y = \sin t$.(5%)

４.Give the vector field $\underset{\sim}{v} = y\underset{\sim}{i} - x\underset{\sim}{j} + z\underset{\sim}{k}$.



(1) Find $\nabla\times\underset{\sim}{v} = ?$ (5%)

(2) Find the line integral $\oint_c \underset{\sim}{v}\cdot\underset{\sim}{t}\, ds = ?$ where c is $x^2 + y^2 = 1$.(圖三)(5%)



(3) Find the surface integral $\iint \nabla\times\underset{\sim}{v}\cdot dS = ?$ (5%)

where S is the hemispherical surface $x^2 + y^2 + z^2 = 1$.(圖四)

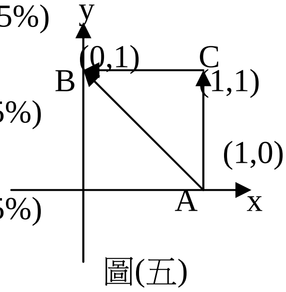
５.Explain why Green's theorem can be special case of Guass theorem and Stokes' theorem.(10%)

６.如圖五

(1) $\int_{AC} \frac{-y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy = ?$ (5%) (4) $\int_{AC} \frac{x}{x^2+y^2}dx + \frac{y}{x^2+y^2}dy = ?$ (5%)

(2) $\int_{CB} \frac{-y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy = ?$ (5%) (5) $\int_{CB} \frac{x}{x^2+y^2}dx + \frac{y}{x^2+y^2}dy = ?$ (5%)

(3) $\int_{AB} \frac{-y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy = ?$ (5%) (6) $\int_{AB} \frac{x}{x^2+y^2}dx + \frac{y}{x^2+y^2}dy = ?$ (5%)



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1.

(1) $\nabla\cdot \underset{\sim}{r}=3$

(2) $x+y=1$, $\oint_C \underset{\sim}{r}\cdot\underset{\sim}{n}\,ds=\oint_c (x\underset{\sim}{i}+y\underset{\sim}{j})\cdot(dy\underset{\sim}{i}-dx\underset{\sim}{j})=\int_0^1 dy=1$

(3) $\iint_{s_1} r\cdot nds=\iiint\nabla\cdot rdV=3V=0\quad(\underset{\sim}{r}\cdot\underset{\sim}{n}=0)$

(4) $\iint_{s_2} r\cdot nds=\iiint\nabla\cdot rdV=3V=\frac{1}{2}$

2.

(1) $r=\sqrt{x^2}$, $\nabla r=\frac{\underset{\sim}{r}}{r}=\frac{x}{\sqrt{x^2}}\underset{\sim}{i}$, $\nabla\cdot(\nabla r)=0$

(2) $\ln r=\frac{1}{2}\ln(x^2+y^2)$, $\nabla\ln r=\frac{1}{2(x^2+y^2)}(x\underset{\sim}{i}+y\underset{\sim}{j})$, $\nabla\cdot\nabla\ln r=0$

(3) $\nabla\frac{1}{r}=\frac{-(x\underset{\sim}{i}+y\underset{\sim}{j}+z\underset{\sim}{k})}{\sqrt{(x^2+y^2+z^2)^2}}$, $\nabla\cdot\nabla\frac{1}{r}=0$

3.

(a) $y=\sqrt{(1-x^2)}$, $y'=\frac{-x}{\sqrt{(1-x^2)}}$, $y''=\frac{-1}{\sqrt{(1-x^2)}}-\frac{x^2}{\sqrt{(1-x^2)^3}}$, $\rho=\frac{(1+(y')^2)^{3/2}}{\|y''\|}=1$

(b) $x=4+\cos t$, $x'=-\sin t$, $x''=-\cos t$, $y=\sin t$, $y'=\cos t$, $y''=-\sin t$, $t=0$

$$\rho=\frac{(x')^2+(y')^2}{(x'y''-x''y')}=1$$

4.

(1) $\nabla\times\underset{\sim}{v}=-2\underset{\sim}{k}$

(2) $\oint_c v\cdot tds=\iint\nabla\times\underset{\sim}{v}dA=-2A=-2\pi$

(3) $\iint\nabla\times\underset{\sim}{v}ds=\iint\nabla\times\underset{\sim}{v}\,dA=-2A=-2\pi$

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5. Green's theorem $\oint P dx + Q dy = \iint (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}) dA$,

Guass theorem $\oint \underset{\sim}{v} \cdot \underset{\sim}{n}\, ds = \iint \nabla \cdot \underset{\sim}{v}\, dxdy \quad \underset{\sim}{v} = (Q, -P)$,

Stokes' theorem $\oint \underset{\sim}{v} \cdot \underset{\sim}{t}\, ds = \iint \nabla \times \underset{\sim}{v} \cdot d\underset{\sim}{A} \quad \underset{\sim}{v} = (P, Q)$

6.

(1) $\int_{AC} \frac{-y}{x^2+y^2} dx + \frac{x}{x^2+y^2} dy = \int_0^1 \frac{1}{1+y^2} dy = \tan^{-1}(1) - \tan^{-1}(0) = \frac{\pi}{4}$

(2) $\int_{CB} \frac{-y}{x^2+y^2} dx + \frac{x}{x^2+y^2} dy = \int_1^0 \frac{-1}{1+x^2} dx = -\tan^{-1}(0) + \tan^{-1}(1) = \frac{\pi}{4}$

(3) $x = 1 - y, \quad dx = -dy$

$$\int_{AB} \frac{-y}{x^2+y^2} dx + \frac{x}{x^2+y^2} dy = \int_1^0 \frac{1}{2(2x^2-2x+1)} dx = \frac{\pi}{2}$$

(4) $\int_{AC} \frac{x}{x^2+y^2} dx + \frac{y}{x^2+y^2} dy = \int_0^1 \frac{y}{1+y^2} dy = \frac{1}{2}(\ln(2) - \ln(1)) = \frac{1}{2}\ln(2)$

(5) $\int_{CB} \frac{x}{x^2+y^2} dx + \frac{y}{x^2+y^2} dy = \int_1^0 \frac{x}{1+x^2} dx = \frac{1}{2}(\ln(1) - \ln(2)) = -\frac{1}{2}\ln(2)$

(6) $y = 1 - x, \quad dy = -dx$

$$\int_{AB} \frac{x}{x^2+y^2} dx + \frac{y}{x^2+y^2} dy = \int_1^0 \frac{-1+2x}{2t^2-2t+1} dt = \frac{1}{2}(\ln(1) - \ln(1)) = 0$$