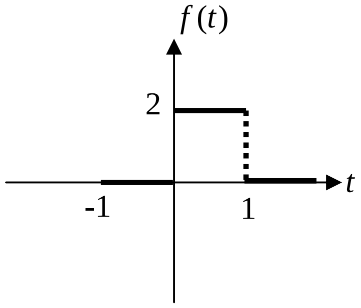
年級：____________　　姓名：__________________　　學號：_______________

國立台灣海洋大學河海工程學系２００４工程數學（三）期末考　Jan.21.2005

Fourier integral



Fourier transform

$\mathcal{F}(f(t))=\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt = F(\omega)$

Hilbert transform

$\mathcal{H}(f(t))=\int_{-\infty}^{\infty} \frac{1}{\pi}\frac{f(u)}{(t-u)}du = H(t)$

(1) Given $f(t)=\begin{cases} 2, & 0<t<1 \\ 0, & otherwise \end{cases}$, decompose $f(t)=f_e(t)+f_0(t)$ and plot $f_e(t)$ and $f_o(t)$. (5%)

(2) Find $F_R(\omega)=\mathcal{F}(f_e(t))$ and $F_I(\omega)=\mathcal{F}(f_o(t))$. (5%)

(3) Find $\mathcal{F}(\mathcal{F}(f_e(t)))$ and $\mathcal{H}(\mathcal{H}(F_R(\omega)))$. (5%)

(4) Given $\operatorname{sgn}(t)=\begin{cases} 1, & t>0 \\ -1, & t<0 \end{cases}$, $S(\omega)=\mathcal{F}(\operatorname{sgn}(t))$

Find $\mathcal{F}(F_R(\omega)*S(\omega))=?$ (5%)

Is it right that $\mathcal{F}(\operatorname{sgn}(-t))=S^*(\omega)$? (*,conjugate) (5%)

(5) Given the convolution, we have $h(t)=f_e(t)*f_e(t)=\int_{-\infty}^{\infty} f_e(u)f_e(t-u)du$. Find $h(0)=?$ (5%)

Given the correlation, we have $h_c(t)=f_e(t)\otimes f_e(t)=\int_{-\infty}^{\infty} f_e(u)f_e(u+t)du$. Find $h_c(0)=?$ (5%)

(6) Find $f_0(t)*f_0(t)$, is it an even function ? (10%)

(7) Find $f_o(t)\otimes f_o(t)$, is it an even function ? (10%)

(8) Determine $\mathcal{F}(f_o(t)\otimes f_o(t))$ and $\mathcal{F}(f_0(t)*f_0(t))$, what is the relation between (6) and (7). (15%)

(9) Find $\int_{-\infty}^{\infty}\frac{1}{\omega^2}\sin^2(\omega)d\omega=?$ (5%)

Fourier series

Choosing a period of 2 for $f(t)=f(t+2)$,

(1) Express the function in terms of real Fourier series (5%) and complex Fourier series (5%).

(2) How to calculate $f'(t)$ in terms of series form ? (5%)

Polar coordinate

Given $u(x,y)=\ln\sqrt{x^2+y^2}=\ln(r)$, please find $\nabla^2 u(x,y)$.

(1) $\nabla^2 u(x,y)=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=?$ (5%)

(2) $\nabla^2 u(x,y)=(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2})=?$ (5%)

where $r^2=x^2+y^2$ and $\theta=\tan^{-1}(y/x)$