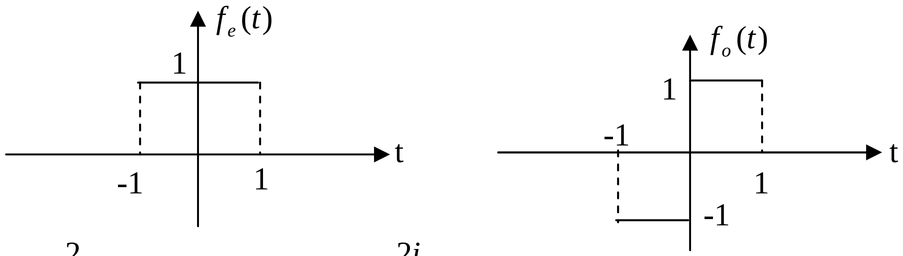
年級：______________ 姓名： ______________________ 學號： _________________

國立台灣海洋大學河海工程學系２００４工程數學（三）期末考解答 Jan.21.2005

Fourier integral

(1)



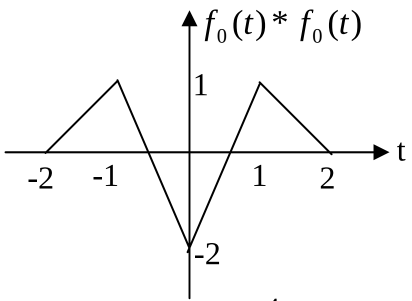
(2) $\mathcal{F}\{f_e(t)\} = \dfrac{2}{\omega}\sin(\omega)$ $\quad$ $\mathcal{F}\{f_o(t)\} = \dfrac{2i}{\omega}(\cos(\omega)-1)$

(3) $2\pi f_e(-t) = 2\pi f_e(t)$ $\quad$ $\mathcal{F}\{\mathcal{F}\{F_R(\omega)\}\} = -F_R(\omega)$

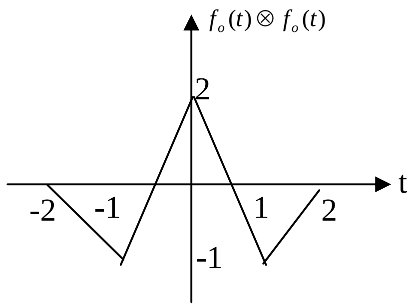
(4) $\mathcal{F}\{F_R(\omega) * S(\omega)\} = (2\pi f_e(-t))(2\pi\,\mathrm{sgn}(-t))$, right

(5) $h(t) = \int_{-1}^{1} 1du = 2$ $\quad$ $h_c(0) = \int_{-1}^{1} 1du = 2$

(6)



(7)



(8) $\mathcal{F}\{f_o(t) \otimes f_o(t)\} = \dfrac{4}{\omega^2}(\cos(\omega)-1)^2$

$\mathcal{F}\{f_0(t) * f_0(t)\} = \dfrac{-4}{\omega^2}(\cos(\omega)-1)^2$, they differ by a minus sign.

(9) $\int_{-\infty}^{\infty} \dfrac{4}{\omega^2}\sin^2(\omega)d\omega = 2\pi\int_{-1}^{1} f_e^{\,2}(t)dt = 2\pi\int_0^1 2dt = 4\pi$ $\quad$ $\therefore \int_{-\infty}^{\infty} \dfrac{\sin^2(\omega)}{\omega^2}d\omega = \pi$

Fourier series

(1) $f(x) = a_0 + \sum_{n=1}^{\infty} a_n\cos(n\pi x) + b_n\sin(n\pi x)$, $a_0 = 1$, $a_n = 0, n \ge 1$, $b_n = \dfrac{-2}{n\pi}(\cos(n\pi)-1)$

$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{in\pi x}$, $c_0 = 1$, $c_n = (\dfrac{i}{n\pi})((-1)^n - 1)$

Using Stokes' theorem formation or Cesaro sum.

Polar coordinate

(1) 0

(2) 0