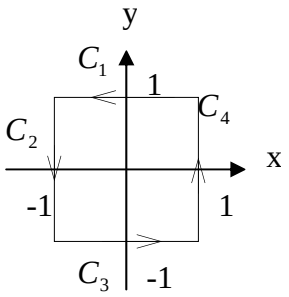
年級：＿＿＿＿＿＿　姓名：＿＿＿＿＿＿＿＿＿＿　學號：＿＿＿＿＿＿＿＿

國立台灣海洋大學河海工程學系２００４工程數學（三）第五次小考解答

1. $\oint_C (\frac{-y}{x^2+y^2})dx + (\frac{x}{x^2+y^2})dy = ?$



$\vec{F} = (\frac{-y}{x^2+y^2}\vec{i} + \frac{x}{x^2+y^2}\vec{j})$

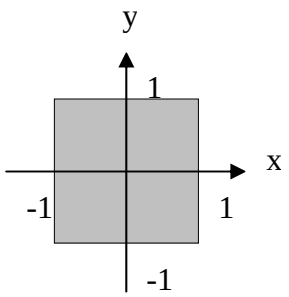
$C_1 \rightarrow x = 1 \sim -1, y = 1 \quad \int_{C_1} \vec{F} \cdot d\vec{r} = \int_1^{-1} \frac{-1}{x^2+1} dx = \frac{\pi}{2}$

$C_2 \rightarrow x = -1, y = 1 \sim -1 \quad \int_{C_2} \vec{F} \cdot d\vec{r} = \int_1^{-1} \frac{-1}{1+y^2} dy = \frac{\pi}{2}$

$C_3 \rightarrow x = -1 \sim 1, y = -1 \quad \int \vec{F} \cdot d\vec{r} = \int_{-1}^{1} \frac{1}{x^2+1} dx = \frac{\pi}{2}$

$C_4 \rightarrow x = 1, y = -1 \sim 1 \quad \int \vec{F} \cdot d\vec{r} = \int_{-1}^{1} \frac{1}{1+y^2} dy = \frac{\pi}{2}$

$\oint_C (\frac{-y}{x^2+y^2})dx + (\frac{x}{x^2+y^2})dy = \int_{C_1+C_2+C_3+C_4} = 2\pi$

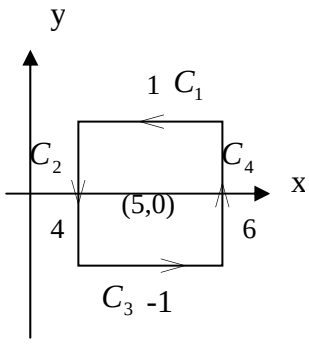


2. $\iint_A [\frac{\partial}{\partial x}(\frac{x}{x^2+y^2}) - \frac{\partial}{\partial y}(\frac{-y}{x^2+y^2})]dA = ?$

A

$\iint_A \frac{x^2+y^2-2x^2}{(x^2+y^2)^2} + \frac{x^2+y^2-2y^2}{(x^2+y^2)^2} dA = 0$

3. $\oint_C (\frac{-y}{x^2+y^2})dx + (\frac{x}{x^2+y^2})dy = ?$

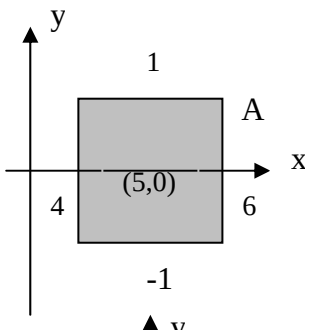


$C_1 \rightarrow \int_4^6 \frac{-1}{x^2+1} dx = \tan^{-1} 6 - \tan^{-1} 4 = C_3$

$C_2 \rightarrow \int_1^{-1} \frac{4}{16+y^2} dy = -2\tan^{-1}\frac{1}{4}$

$C_4 \rightarrow \int_{-1}^{1} \frac{6}{36+y^2} dy = 2\tan^{-1}(\frac{1}{6})$

$\oint_C (\frac{-y}{x^2+y^2})dx + (\frac{x}{x^2+y^2})dy = -\pi + \pi = 0$



4. $\iint_A [\frac{\partial}{\partial x}(\frac{x}{x^2+y^2}) - \frac{\partial}{\partial y}(\frac{-y}{x^2+y^2})]dA = ?$

$\iint_A \frac{x^2+y^2-2x^2}{(x^2+y^2)^2} + \frac{x^2+y^2-2y^2}{(x^2+y^2)^2} dA = 0$

5. $\iint_A [\frac{\partial}{\partial x}(\frac{x}{x^2+y^2}) - \frac{\partial}{\partial y}(\frac{-y}{x^2+y^2})]dA = ?$

0