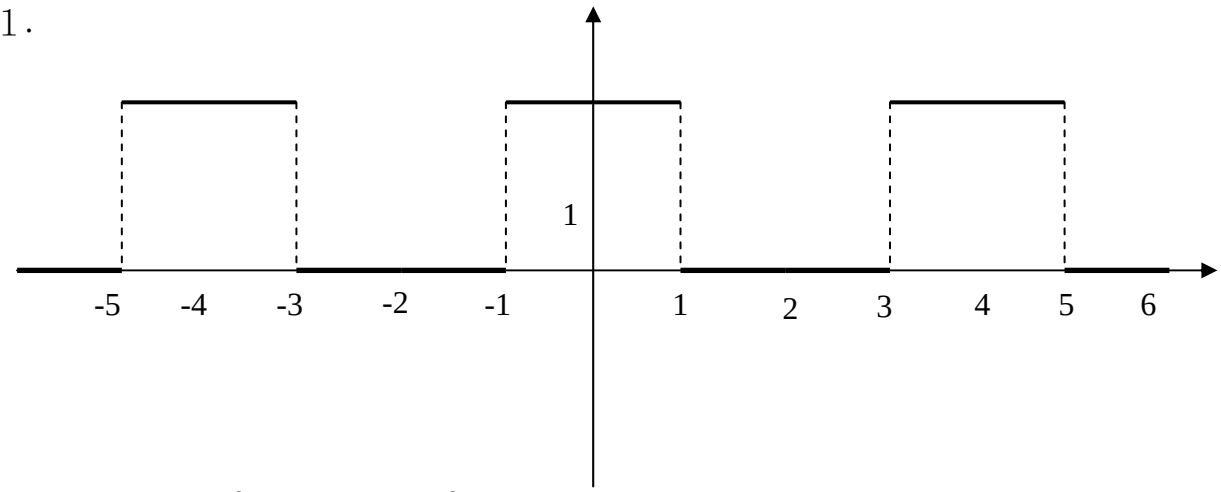
年級：＿＿＿＿＿＿　姓名：＿＿＿＿＿＿＿＿＿＿　學號：＿＿＿＿＿＿＿＿

國立台灣海洋大學河海工程學系２００４工程數學（三）第六次小考解答

1.



Please expand the fuction in term of Fourier series.

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(\frac{n\pi}{2}x) + b_n \sin(\frac{n\pi}{2}x)$$

$$a_0 = \frac{1}{4}\int_{-1}^{1} f(x)dx = \frac{1}{2}\int_{0}^{1} dx = \frac{1}{2}$$

$$a_n = \frac{1}{2}\int_{-1}^{1} f(x)\cos(\frac{n\pi}{2}x)dx = \int_{0}^{1}\cos(\frac{n\pi}{2}x)dx = \frac{2}{n\pi}\sin(\frac{n\pi}{2})$$

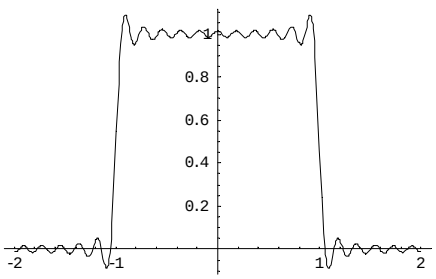
$f(x)$ 爲偶函數，所以 $b_n = 0$

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty}\frac{2}{n\pi}\sin(\frac{n\pi}{2})\cos(\frac{n\pi}{2}x)$$

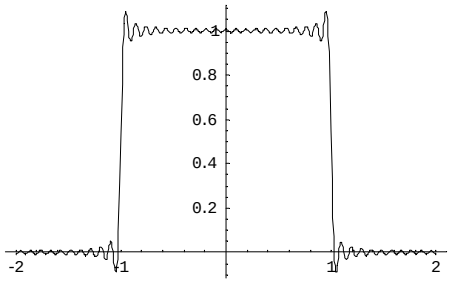
$$n = 2k \qquad \sin(\frac{k\pi}{2}) = 0$$

$$n = 2k+1 \quad \sin[(\frac{2k+1}{2})\pi] = (-1)^k$$

$$f(x) = \frac{1}{2} + \sum_{k=0}^{\infty}\frac{2}{(2k+1)\pi}(-1)^k\cos[(\frac{2k+1}{2})\pi x]$$



$$f = \frac{1}{2} + \text{Sum}\left[\frac{2}{k(2k+1)\pi}(-1)^k\cos\left[\frac{(2k+1)\pi}{2}x\right], \{k, 0, 10\}\right]$$



$$f = \frac{1}{2} + \text{Sum}\left[\frac{2}{k(2k+1)\pi}(-1)^k\cos\left[\frac{(2k+1)\pi}{2}x\right], \{k, 0, 20\}\right]$$