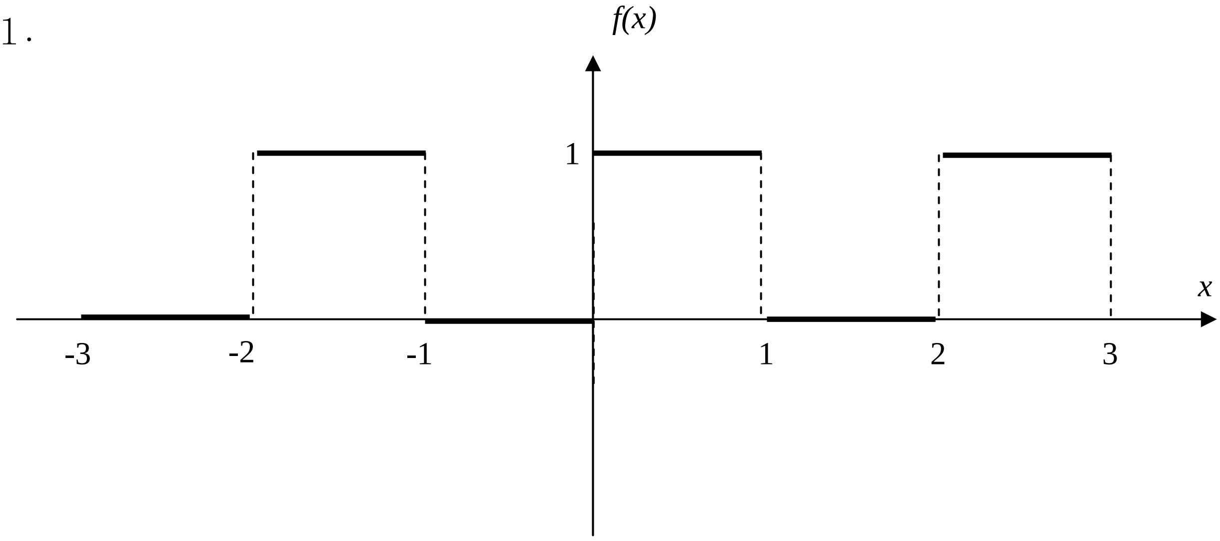
年級：______________　　姓名：____________________　　學號：________________

國立台灣海洋大學河海工程學系２００４工程數學（三）第七次小考解答

1.



(1) Decompose the function into $y_e(x)$ and $y_o(x)$ and plot $y_e(x)$ and $y_o(x)$.

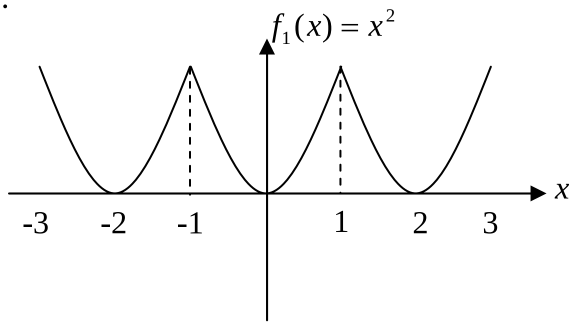
(2) Expand $y_e(x)$ in terms of Fourier series.

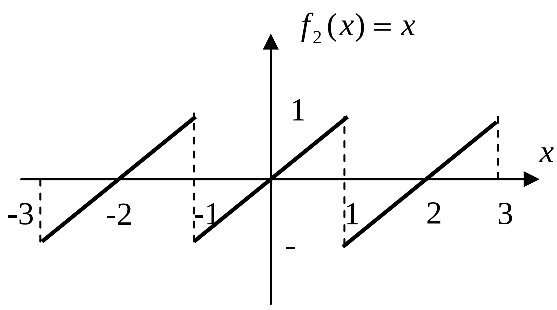
(3) Expand $y_o(x)$ in terms of Fourier series.

(4) Expand $y(x)$ in terms of Fourier series.

(5) If we look function to be period of 4, expand $y(x)$ and compare the one of the period 2.

2.





(1) Expand $f_1(x)$ into Fourier series.

(2) Expand $f_2(x)$ into Fourier series.

(3) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

(4) $\sum_{n=1}^{\infty} \frac{1}{n^4}$

(5) $\sum_{n=1}^{\infty} \frac{1}{n^6}$

(Hint:Parseval's theorem)

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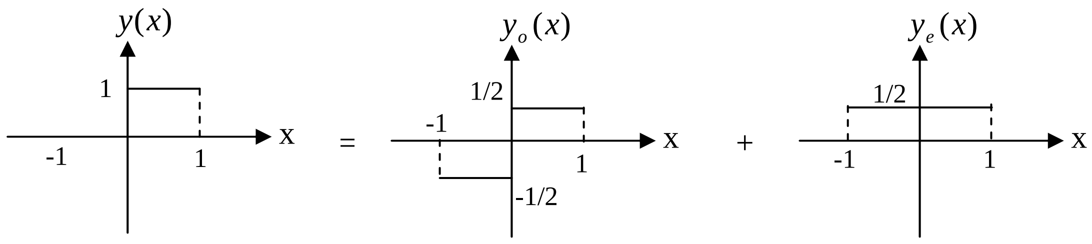
1.

(1)



(2) $y_e(x)=\frac{1}{2}$

(3) $y_o(x)=\sum_{n=1}^{\infty}\frac{1}{n\pi}(1-(-1)^n)\sin(n\pi x)=\sum_{k=0}^{\infty}\frac{2}{(2k+1)\pi}\sin((2k+1)\pi x)$

(4) $y(x)=\frac{1}{2}+\sum_{n=1}^{\infty}\frac{1}{n\pi}(1-(-1)^n)\sin(n\pi x)=\frac{1}{2}+\sum_{k=0}^{\infty}\frac{2}{(2k+1)\pi}\sin((2k+1)\pi x)$

(5) $a_0=\frac{1}{4}[\int_{-2}^{-1}dx+\int_0^1 dx]=\frac{1}{2}$, $\quad a_n=\frac{1}{2}[\int_{-2}^{-1}\cos(\frac{n\pi}{2}x)dx+\int_0^1\cos(\frac{n\pi}{2}x)dx]=0$

$b_n=\frac{1}{2}[\int_{-2}^{-1}\sin(\frac{n\pi}{2}x)dx+\int_0^1\sin(\frac{n\pi}{2}x)dx]=\frac{1}{n\pi}[\cos(n\pi)+1-2\cos(\frac{n\pi}{2})]$

$y(x)=\frac{1}{2}+\sum_{k=0}^{\infty}\frac{2}{(2k+1)\pi}\sin((2k+1)\pi x)$

T=4 與 T=2 做傅立業展開其結果相同。

2.

(1) $a_0=\frac{1}{2}\int_{-1}^{1}x^2dx=\frac{1}{3}$, $\quad a_n=\int_{-1}^{1}x^2\cos(n\pi x)dx=(\frac{1}{n\pi})^2 4\cos(n\pi)$, $\quad b_n=\int_{-1}^{1}x^2\sin(n\pi x)dx=0$

$f_1(x)=\frac{1}{3}+\sum_{n=1}^{\infty}\frac{4}{n^2\pi^2}(-1)^n\cos(n\pi x)$

(2) $a_0=\frac{1}{2}\int_{-1}^{1}xdx=0$, $\quad a_n=\int_{-1}^{1}x\cos(n\pi x)dx=0$, $\quad b_n=\int_{-1}^{1}x\sin(n\pi x)dx=\frac{-2}{n\pi}\cos(n\pi)$

$f_2(x)=\sum_{n=1}^{\infty}\frac{-2}{n\pi}(-1)^n\sin(n\pi x)$

(3) $f_1(1)=\frac{1}{3}+\sum_{n=1}^{\infty}\frac{4}{n^2\pi^2}=1$, $\qquad \sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}\approx 1.63498$

(4) $\frac{1}{2}\int_{-1}^{1}x^4dx=\frac{1}{9}+\sum_{n=1}^{\infty}\frac{8}{n^4\pi^4}$, $\qquad \sum_{n=1}^{\infty}\frac{1}{n^4}=\frac{\pi^4}{90}\approx 1.08232$

(5) $\int_0^x f_1(x)dx=\int_0^x(\frac{1}{3}+\sum_{n=1}^{\infty}\frac{4}{n^2\pi^2}(-1)^n\cos(n\pi x))dx$

$\frac{1}{2}\int_{-1}^{1}(\frac{x^3}{3}-\frac{x}{3})^2dx=\sum_{n=1}^{\infty}\frac{8}{n^6\pi^6}$, $\quad \sum_{n=1}^{\infty}\frac{1}{n^6}=\frac{\pi^6}{945}\approx 1.01734$