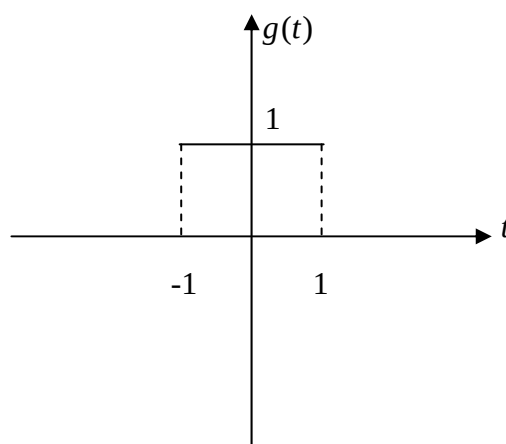
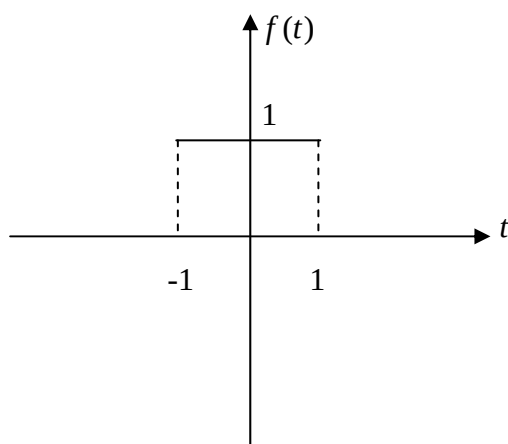


國立台灣海洋大學河海工程學系 2004 工程數學（三）第九次小考解答

1.



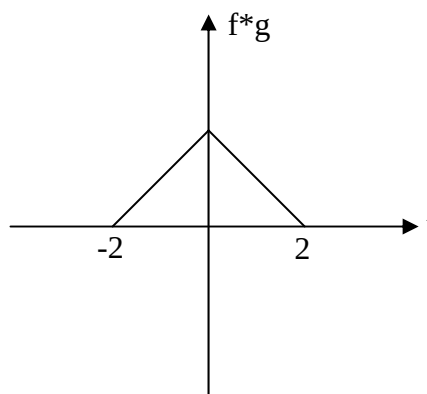
(1) Find $F(\omega) = ?$

(2) Find $f(t) * g(t) = ?$

(3) Find $\int_{-\infty}^{\infty} |F(\omega)|^2 d\omega = ?$

(4) Find $\int_{-\infty}^{\infty} f^2(t) dt = ?$

(5) Find $\int_{-\infty}^{\infty} (f(t) * g(t))^2 dt = ?$



$$(1) F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt = \frac{2}{\omega} \sin(\omega)$$

$$(2) f(t) * g(t) = \int_{-\infty}^{\infty} f(u) g(t-u) du = \int_{-\infty}^{\infty} du = \begin{cases} 0 < t < 2, & 2-t \\ -2 < t < 0 & 2+t \\ \text{other} & 0 \end{cases}$$

$$(3) \int_{-\infty}^{\infty} (F(\omega))^2 d\omega = 2\pi \int_{-1}^1 f^2(t) dt = 4\pi$$

$$(4) \int_{-\infty}^{\infty} f^2(t) dt = \int_{-1}^1 dt = 2$$

$$(5) \int_{-\infty}^{\infty} (f(t) * g(t))^2 dt = \int_0^2 (2-t)^2 dt + \int_{-2}^0 (2+t)^2 dt = \frac{16}{3}$$

$$(6) \mathcal{F}(f(t) * g(t)) = \frac{4}{\omega^2} \sin^2(\omega)$$