班級：______________學號：_______________姓名：_______________

海洋大學河海工程學系 2005 工程數學(四)期末考(Open Book)

1. Find the possible functions $X_n(x)$ such that

$$X_n''(x) = -\lambda X_n(x),$$
$$X_n'(0) = X_n'(\pi) = 0$$

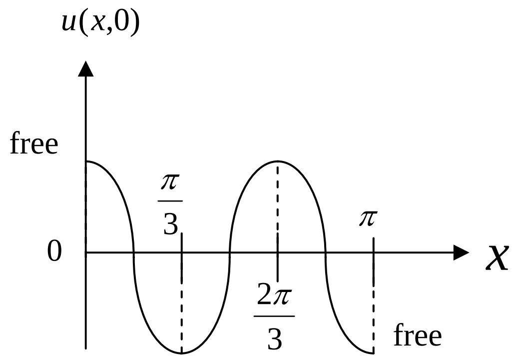
Also, find the eigenvalues λ_n. (20%)

Ans: $X_n(x) = \cos(nx)$

$$\lambda_n = n^2$$

2. A free-free string with a length π

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$$
$$u(x,0) = \cos(3x)$$
$$\dot{u}(x,0) = 0$$
$$u_x(0,t) = u_x(\pi,t) = 0$$



find $u(\frac{\pi}{2},\pi) = ?$

Using (1) Diamond rule (10%)
(2) Image method (10%)
(3) Series solution (10%)

Ans: $u(x,t) = \cos(3x)\cos(3t)$

(1) 0
(2) 0
(3) 0

3. Explain: (1) Laplace equation (5%)
(2) Wave equation (5%)
(3) Heat equation (5%)
(4) Characteristic line (5%)
(5) D'Alembert solution (5%)

Ans: (1) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

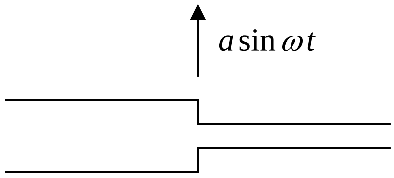
(2) $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$

(3) $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$

(4) $x \pm ct$

(5) $u(x,t) = \frac{1}{2}[\phi(x+ct) - \phi(x-ct)] + \frac{1}{2c}\int_{x-ct}^{x+ct} \psi(\tau)d\tau$

4. Solve the PDE: $u_{tt} = \begin{cases} 4u_{xx}, & x<0, t>0 \\ 1u_{xx}, & x>0, t>0 \end{cases}$

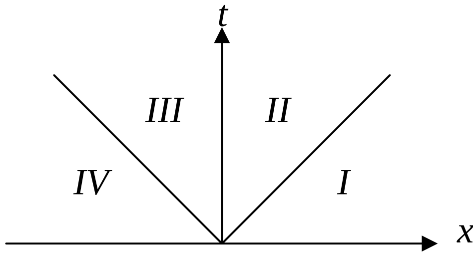


I.C.: $u(x,0) = \dot{u}(x,0) = 0$

At the interface, we apply the force

$$u_x(0^+,t) - u_x(0^-,t) = a\sin\omega t. \quad (20\%)$$

Hint: Using diamond rule



Ans: $u^I(x,t) = 0$

$$u^{II}(x,t) = \frac{2a}{3\omega}\cos[\omega(t-x)] - \frac{2a}{3\omega}$$

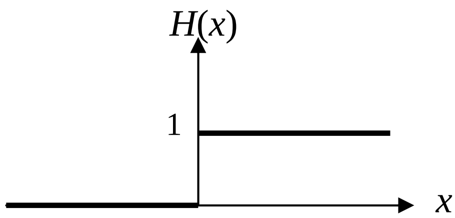
$$u^{III}(x,t) = \frac{2a}{3\omega}\cos[\omega(t+\frac{x}{2})] - \frac{2a}{3\omega}$$

$$u^{IV}(x,t) = 0$$

5. Solve $u_{tt} = u_{xx}$

$u(x,0) = 0$

$\dot{u}(x,0) = \frac{1}{a}[H(x+a) - H(x-a)]$



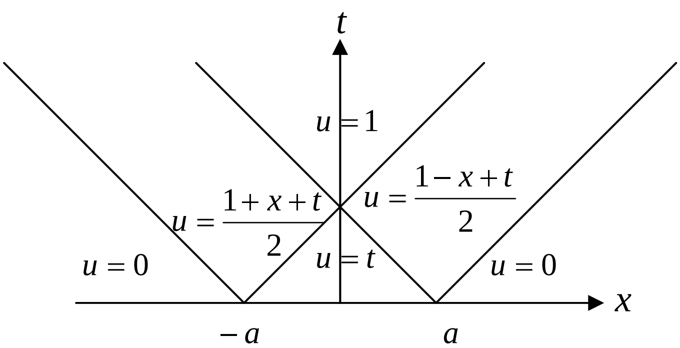
$$H(x) = \begin{cases} 1, & x>0 \\ 0, & x<0 \end{cases}$$

(1) Solve $u(x,t)$ for $a=1$. (10%)

(2) Solve $u(x,t)$ for $a \to 0$. (10%)

Ans:

$a = 1$



$a \to 0$