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ON FICTITIOUS FREQUENCIES USING CIRCULANTS FOR RADIATION PROBLEMS OF A CYLINDER

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Introduction

Integral equation has been used to solve exterior acoustic problems (radiation and scattering) for many years. It is well known that fictitious (irregular) frequencies stem from the numerical resonance instead of physical resonance if integral representation for the solution is assumed. Many researchers claimed that integral solution does not have a unique solution at the natural frequencies of an associated interior problem [1-11]. However, their conclusions on the "associated interior problem" are not correct. Chen in [12, 13, 14] confirmed the conclusion that the positions of fictitious frequencies are independent of the boundary conditions once the method is chosen by using the dual series model. The roles of dual formulations can be found in [15]. To demonstrate why fictitious frequencies occur, Chen and Hong [12] and Chen *et al.* [13] showed that they depend on the kernels in the integral representation for the solution by using one and two dimensional examples, respectively. Also, numerical experiments using the dual BEM program were performed and the results matched the analytical solutions well [13]. From the numerical point of view, this nonunique problem can be seen as the indeterminate form of zero divided by zero. Since the L'hospital's rule can be employed analytically, no fictitious frequencies (or wave number) occur. However, L'Hospital's rule can not be applied in the numerical calculation for computer.

In this paper, a circular boundary is discretized into finite-length circular arcs and the influence matrices results in circulants [19, 20, 21, 22] due to the circular symmetry. The circulants for the influence matrices corresponding to the four kernel functions in the dual formulation are constructed to verify the conclusion in [14]. The relations of the influence matrices between the interior and exterior acoustic problems are also examined. The positions of fictitious frequencies for the exterior problems using only the UT (singular integral equation) or LM (hypersingular integral equation) formulation are derived by transforming the finite-dimensional space into continuous system. Two examples, including the Dirichlet and Neuman problems, are illustrated to show the mechanism of fictitious frequencies in radiation problems of a cylinder. After the kernels are expressed in degenerate form using dual series model, the influence matrices can be constructed. Since a circular domain is considered, the four matrices result in circulants [19, 20, 21, 22] such that the spectral properties can be investigated analytically. The eigenvalues for the circulants are found analytically using the similar property and the determinant for the matrices can be obtained easily [19, 20, 21, 22, 24] Numerical results using the dual BEM program are verified in comparision with the analytical solutions. It is shown that the type of boundary condition, Dirichlet or Neumann, can not change the positions where fictitious frequencies occur once the integral representation for the solution is chosen. Based on the theoretical proof [13, 14] for continuous system and present study using circulants for a discrete system, some misleading statements can be clarified.

Dual integral formulation for a two-dimensional exterior acoustic radiation problem

The governing equation for an exterior acoustic problem is the Helmholtz equation as follows:

$$(\nabla^2 + k^2)u(x_1, x_2) = 0, \ \ (x_1, x_2) \in D,$$

where ∇^2 is the Laplacian operator, D is the domain of the cavity and k is the wave number, which is angular frequency over the speed of sound. For simplicity, radiation problem is considered only. The boundary conditions can be either the Neumann or Dirichlet type.

Based on the dual formulation [16, 17], the dual equations for the boundary points are

$$\pi u(x) = C.P.V. \int_B T(s,x)u(s)dB(s) - R.P.V. \int_B U(s,x)t(s)dB(s), \ x \in B \tag{1}$$

$$\pi t(x) = H.P.V. \int_B M(s,x)u(s)dB(s) - C.P.V. \int_B L(s,x)t(s)dB(s), \ x \in B \tag{2}$$

where $C.P.V.$, $R.P.V.$ and $H.P.V.$ denote the Cauchy principal value, the Riemann principal value and the Hadamard principal value, $t(s) = \frac{\partial u(s)}{\partial n_s}$, B denotes the boundary enclosing D and the explicit forms of the four kernels, U, T, L and M, can be found in [16].

Relations of the influence matrices between interior and exterior problems using the dual BEM

The linear algebraic equations for an interior problem discretized from the dual boundary integral equations can be written as

$$[T^i_{pq}]\{u_q\} = [U^i_{pq}]\{t_q\} \tag{3}$$

$$[M^i_{pq}]\{u_q\} = [L^i_{pq}]\{t_q\}, \tag{4}$$

where the superscript "i" denotes the interior problem, $\{u_q\}$ and $\{t_q\}$ are the boundary potential and flux, and the subscripts p and q correspond to the labels of the collocation element and integration element, respectively. The influence coefficients of the four square matrices $[U], [T], [L]$ and $[M]$ can be represented as

$$U^i_{pq} = R.P.V. \int_{B_q} U(s_q, x_p)dB(s_q) \tag{5}$$

$$T^i_{pq} = -\pi\delta_{pq} + C.P.V. \int_{B_q} T(s_q, x_p)dB(s_q) \tag{6}$$

$$L^i_{pq} = \pi\delta_{pq} + C.P.V. \int_{B_q} L(s_q, x_p)dB(s_q) \tag{7}$$

$$M^i_{pq} = H.P.V. \int_{B_q} M(s_q, x_p)dB(s_q), \tag{8}$$

where B_q denotes the q^{th} element and $\delta_{pq} = 1$ if $p = q$; otherwise it is zero.

For the exterior problem, we have

$$[T^e_{pq}]\{u_q\} = [U^e_{pq}]\{t_q\} \tag{9}$$

$$[M^e_{pq}]\{u_q\} = [L^e_{pq}]\{t_q\}. \tag{10}$$

where the superscript "e" denotes the exterior problem. According to the dependence of the outnormal vectors in these four kernel functions for the interior and exterior problems, their relationship can be easily found as shown below [17]:

$$U^i_{pq} = U^e_{pq} \tag{11}$$

$$M^i_{pq} = M^e_{pq} \tag{12}$$

$$T^i_{pq} = \begin{cases} -T^e_{pq}, & if\ p \neq q, \\ T^e_{pq}, & if\ p = q \end{cases} \tag{13}$$

$$L^i_{pq} = \begin{cases} -L^e_{pq}, & if\ p \neq q, \\ L^e_{pq}, & if\ p = q. \end{cases} \tag{14}$$

Based on the relations in Eqs.(11) $\sim$ (14), the dual BEM program [16] can be easily extended to solve for exterior problems. In order to compare with the analytical solutions, a circular domain is considered. Without loss of generality, the conclusion of the present paper can be applied to problems with general geometries and will be published in the followinfg paper. The absolute value for the determinant of the eight matrices, $U^i_{pq}, T^i_{pq}, L^i_{pq}, M^i_{pq}$ $U^e_{pq}, T^e_{pq}, L^e_{pq}$ and M^e_{pq} versus the wave number k are ploted in Fig.1 using the dual BEM program [16]. The poles or eigenvalues occur at the local minimum. It is found that the characteristic values for the four kernels, $U^i_{pq}, L^i_{pq}, U^e_{pq}$, and T^e_{pq}, are equal to the eigenvalues of the associated interior Dirichlet problem as shown in Table 1. On the other hand, the four kernels $T^i_{pq}, M^i_{pq}, L^e_{pq}$, and M^e_{pq} have the same characteristic values which are the eigenvalues of the interior associated Neumann problem as shown in Table 2.

Table 1 Characteristic solutions for the Helmholtz equation with the Dirichlet boundary conditions

No. (n)	eigenvalues (k_n)	eigen equation	eigenmode $u_n(r,\theta)$	multiplicity
1	2.4048(2.4070)	$J_0(ka) = 0$	$J_0(2.4048r)$	1
2,3	3.8317(3.8342)	$J_1(ka) = 0$	$J_1(3.8317r)e^{\pm i\theta}$	2
4,5	5.1356(5.1388)	$J_2(ka) = 0$	$J_2(5.1356r)e^{\pm i2\theta}$	2
6	5.5201(5.5223)	$J_0(ka) = 0$	$J_0(5.5201r)$	1

Note that data in parenthesis are obtained by the dual BEM.

Table 2 Characteristic solutions for the Helmholtz equation with the Neumann boundary conditions

No. (n)	eigenvalues (k_n)	eigen equation	eigenmode $u_n(r,\theta)$	multiplicity
1	0.0000(0.0000)	$J_0'(ka) = 0$	$J_0(0.0000r)$	1
2,3	1.8412(1.8436)	$J_1'(ka) = 0$	$J_1(1.8412r)e^{\pm i\theta}$	2
4,5	3.0542(3.0586)	$J_2'(ka) = 0$	$J_2(3.0542r)e^{\pm i2\theta}$	2
6	3.8317(3.8364)	$J_0'(ka) = 0$	$J_0(3.8317r)$	1

Note that data in parenthesis are obtained by the dual BEM.

Circulant matrices for exterior problems in the dual BEM

By using the two bases $J_m(kx)$ and $Y_m(kx)$, we can decompose the two-dimensional kernel functions into

$$U(s,x) = \begin{cases} U^i(\theta,0) = \sum_{m=-\infty}^{\infty} \frac{\pi}{2}[-iJ_m(kR)+Y_m(kR)]J_m(k\rho)cos(k\theta), & R>\rho \\ U^e(\theta,0) = \sum_{m=-\infty}^{\infty} \frac{\pi}{2}[-iJ_m(k\rho)+Y_m(k\rho)]J_m(kR)cos(k\theta), & R<\rho \end{cases} \quad (15)$$

$$T(s,x) = \begin{cases} T^i(\theta,0) = \sum_{m=-\infty}^{\infty} \frac{\pi k}{2}[-iJ'_m(kR)+Y'_m(kR)]J_m(k\rho)cos(k\theta), & R>\rho \\ T^e(\theta,0) = \sum_{m=-\infty}^{\infty} \frac{\pi k}{2}[-iJ_m(k\rho)+Y_m(k\rho)]J'_m(kR)cos(k\theta), & R<\rho \end{cases} \quad (16)$$

$$L(s,x) = \begin{cases} L^i(\theta,0) = \sum_{m=-\infty}^{\infty} \frac{\pi k}{2}[-iJ_m(kR)+Y_m(kR)]J'_m(k\rho)cos(k\theta), & R>\rho \\ L^e(\theta,0) = \sum_{m=-\infty}^{\infty} \frac{\pi k}{2}[-iJ'_m(k\rho)+Y'_m(k\rho)]J_m(kR)cos(k\theta), & R<\rho \end{cases} \quad (17)$$

$$M(s,x) = \begin{cases} M^i(\theta,0) = \sum_{m=-\infty}^{\infty} \frac{\pi k^2}{2}[-iJ'_m(kR)+Y'_m(kR)]J'_m(k\rho)cos(k\theta), & R>\rho \\ M^e(\theta,0) = \sum_{m=-\infty}^{\infty} \frac{\pi k^2}{2}[-iJ'_m(k\rho)+Y'_m(k\rho)]J'_m(kR)cos(k\theta), & R<\rho \end{cases} \quad (18)$$

where J_m and Y_m are the mth order Bessel functions of the first and second kind, respectively, $x=(\rho,0)$ and $s=(R,\theta)$ in polar coordinate. For a problem with a circular domain, the circular boundary can be discretized into $2N$ arcs with equal length. Based on the circular symmetry, the influence matrices for the discrete system are found to be circulants. All the influence matrices can be expressed as shown below.

$$[U^i] = \begin{bmatrix} a_0 & a_1 & a_2 & \cdots & a_{2N-2} & a_{2N-1} \\ a_{2N-1} & a_0 & a_1 & \cdots & a_{2N-3} & a_{2N-2} \\ a_{2N-2} & a_{2N-1} & a_0 & \cdots & a_{2N-4} & a_{2N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_1 & a_2 & a_3 & \cdots & a_{2N-1} & a_0 \end{bmatrix} \quad (19)$$

$$[T^i] = \begin{bmatrix} b_0 & b_1 & b_2 & \cdots & b_{2N-2} & b_{2N-1} \\ b_{2N-1} & b_0 & b_1 & \cdots & b_{2N-3} & b_{2N-2} \\ b_{2N-2} & b_{2N-1} & b_0 & \cdots & b_{2N-4} & b_{2N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ b_1 & b_2 & b_3 & \cdots & b_{2N-1} & b_0 \end{bmatrix} \quad (20)$$

$$[L^i] = \begin{bmatrix} c_0 & c_1 & c_2 & \cdots & c_{2N-2} & c_{2N-1} \\ c_{2N-1} & c_0 & c_1 & \cdots & c_{2N-3} & c_{2N-2} \\ c_{2N-2} & c_{2N-1} & c_0 & \cdots & c_{2N-4} & c_{2N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ c_1 & c_2 & c_3 & \cdots & c_{2N-1} & c_0 \end{bmatrix} \quad (21)$$

$$[M^i] = \begin{bmatrix} d_0 & d_1 & d_2 & \cdots & d_{2N-2} & d_{2N-1} \\ d_{2N-1} & d_0 & d_1 & \cdots & d_{2N-3} & d_{2N-2} \\ d_{2N-2} & d_{2N-1} & d_0 & \cdots & d_{2N-4} & d_{2N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ d_1 & d_2 & d_3 & \cdots & d_{2N-1} & d_0 \end{bmatrix} \quad (22)$$

$$[U^e] = \begin{bmatrix} p_0 & p_1 & p_2 & \cdots & p_{2N-2} & p_{2N-1} \\ p_{2N-1} & p_0 & p_1 & \cdots & p_{2N-3} & p_{2N-2} \\ p_{2N-2} & p_{2N-1} & p_0 & \cdots & p_{2N-4} & p_{2N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ p_1 & p_2 & p_3 & \cdots & p_{2N-1} & p_0 \end{bmatrix} \tag{23}$$

$$[T^e] = \begin{bmatrix} q_0 & q_1 & q_2 & \cdots & q_{2N-2} & q_{2N-1} \\ q_{2N-1} & q_0 & q_1 & \cdots & q_{2N-3} & q_{2N-2} \\ q_{2N-2} & q_{2N-1} & q_0 & \cdots & q_{2N-4} & q_{2N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ q_1 & q_2 & q_3 & \cdots & q_{2N-1} & q_0 \end{bmatrix} \tag{24}$$

$$[L^e] = \begin{bmatrix} v_0 & v_1 & v_2 & \cdots & v_{2N-2} & v_{2N-1} \\ v_{2N-1} & v_0 & v_1 & \cdots & v_{2N-3} & v_{2N-2} \\ v_{2N-2} & v_{2N-1} & v_0 & \cdots & v_{2N-4} & v_{2N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ v_1 & v_2 & v_3 & \cdots & v_{2N-1} & v_0 \end{bmatrix} \tag{25}$$

$$[M^e] = \begin{bmatrix} w_0 & w_1 & w_2 & \cdots & w_{2N-2} & w_{2N-1} \\ w_{2N-1} & w_0 & w_1 & \cdots & w_{2N-3} & w_{2N-2} \\ w_{2N-2} & w_{2N-1} & w_0 & \cdots & w_{2N-4} & w_{2N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ w_1 & w_2 & w_3 & \cdots & w_{2N-1} & w_0 \end{bmatrix} \tag{26}$$

where

$$a_m = \int_{(m-\frac{1}{2})\Delta\theta}^{(m+\frac{1}{2})\Delta\theta} U^e(\theta,0)\rho\, d\theta \approx U^e(\theta_m,0)\rho\,\Delta\theta, \quad m = 0,1,2,\cdots,2N-1 \tag{27}$$

$$b_m = \int_{(m-\frac{1}{2})\Delta\theta}^{(m+\frac{1}{2})\Delta\theta} T^e(\theta,0)\rho\, d\theta \approx T^e(\theta_m,0)\rho\,\Delta\theta, \quad m = 0,1,2,\cdots,2N-1 \tag{28}$$

$$c_m = \int_{(m-\frac{1}{2})\Delta\theta}^{(m+\frac{1}{2})\Delta\theta} L^e(\theta,0)\rho\, d\theta \approx L^e(\theta_m,0)\rho\,\Delta\theta, \quad m = 0,1,2,\cdots,2N-1 \tag{29}$$

$$d_m = \int_{(m-\frac{1}{2})\Delta\theta}^{(m+\frac{1}{2})\Delta\theta} M^e(\theta,0)\rho\, d\theta \approx M^e(\theta_m,0)\rho\,\Delta\theta, \quad m = 0,1,2,\cdots,2N-1 \tag{30}$$

$$p_m = \int_{(m-\frac{1}{2})\Delta\theta}^{(m+\frac{1}{2})\Delta\theta} U^i(\theta,0)\rho\, d\theta \approx U^i(\theta_m,0)\rho\,\Delta\theta, \quad m = 0,1,2,\cdots,2N-1 \tag{31}$$

$$q_m = \int_{(m-\frac{1}{2})\Delta\theta}^{(m+\frac{1}{2})\Delta\theta} T^i(\theta,0)\rho\, d\theta \approx T^i(\theta_m,0)\rho\,\Delta\theta, \quad m = 0,1,2,\cdots,2N-1 \tag{32}$$

$$v_m = \int_{(m-\frac{1}{2})\Delta\theta}^{(m+\frac{1}{2})\Delta\theta} L^i(\theta,0)\rho\, d\theta \approx L^i(\theta_m,0)\rho\,\Delta\theta, \quad m=0,1,2,\cdots,2N-1 \tag{33}$$

$$w_m = \int_{(m-\frac{1}{2})\Delta\theta}^{(m+\frac{1}{2})\Delta\theta} M^i(\theta,0)\rho\, d\theta \approx M^i(\theta_m,0)\rho\,\Delta\theta, \quad m=0,1,2,\cdots,2N-1 \tag{34}$$

in which $\theta_m = m\Delta\theta$ and $\Delta\theta = \frac{2\pi}{2N}$. Based on the circular symmetry, it is easy to find that all the influence matrices in Eqs.(19) $\sim$ (26) are symmetric circulants. By using the similar properties for all the eight matrices with respect to circulants, we have

$$det[U^i] = \lambda_0(\lambda_1\lambda_2\cdots\lambda_{N-1})^2\lambda_N \tag{35}$$

$$det[U^e] = \lambda_0(\lambda_1\lambda_2\cdots\lambda_{N-1})^2\lambda_N \tag{36}$$

$$det[T^e] = \mu_0(\mu_1\mu_2\cdots\mu_{N-1})^2\mu_N \tag{37}$$

$$det[L^i] = \mu_0(\mu_1\mu_2\cdots\mu_{N-1})^2\mu_N \tag{38}$$

$$det[T^i] = \nu_0(\nu_1\nu_2\cdots\nu_{N-1})^2\nu_N \tag{39}$$

$$det[L^e] = \nu_0(\nu_1\nu_2\cdots\nu_{N-1})^2\nu_N \tag{40}$$

$$det[M^i] = \kappa_0(\kappa_1\kappa_2\cdots\kappa_{N-1})^2\kappa_N \tag{41}$$

$$det[M^e] = \kappa_0(\kappa_1\kappa_2\cdots\kappa_{N-1})^2\kappa_N \tag{42}$$

where

$$\lambda_\ell = \pi^2\rho\,(-iJ_\ell(k\rho) + Y_\ell(k\rho))J_\ell(k\rho), \ \ \ell = 0, \pm1, \cdots, \pm(N-1), N. \tag{43}$$

$$\mu_\ell = \pi^2 k\rho\,(-iJ'_\ell(k\rho) + Y'_\ell(k\rho))J_\ell(k\rho), \ \ \ell = 0, \pm1, \cdots, \pm(N-1), N. \tag{44}$$

$$\nu_\ell = \pi^2 k\rho\,(-iJ_\ell(k\rho) + Y_\ell(k\rho))J'_\ell(k\rho), \ \ \ell = 0, \pm1, \cdots, \pm(N-1), N. \tag{45}$$

$$\kappa_\ell = \pi^2 k^2\rho\,(-iJ'_\ell(k\rho) + Y'_\ell(k\rho))J'_\ell(k\rho), \ \ \ell = 0, \pm1, \cdots, \pm(N-1), N. \tag{46}$$

According to Eqs.(35) $\sim$ (42), the corresponding spectral properties are shown in Table 3. The fictitious solution for the exterior Helmhotlz problem is shown in Tables 4 and 5 using the UT and LM formulation, respectively.

Table 3 The spectral properties of the four influence matrices for interior and exterior domains in the dual formulation

$J_m(ka) = 0$	U^i	U^e	T^e	L^i
$J'_m(ka) = 0$	M^i	M^e	T^i	L^e

Table 4 Fictitious solutions for the exterior Helmholtz equation using the UT formulation

No. (n)	fictitious values (k_n)	fictitious equation	multiplicity
1	2.4048(2.4070)	$J_0(ka) = 0$	1
2,3	3.8317(3.8342)	$J_1(ka) = 0$	2
4,5	5.1356(5.1388)	$J_2(ka) = 0$	2
6	5.5201(5.5223)	$J_0(ka) = 0$	1

Note that data in parenthesis are obtained by dual BEM.

Table 5 Fictitious solutions for the exterior Helmholtz equation using the LM formulation

No. (n)	eigenvalues (k_n)	eigen equation	multiplicity
1	0.0000(0.0000)	$J_0'(ka)=0$	1
2,3	1.8412(1.8436)	$J_1'(ka)=0$	2
4,5	3.0542(3.0586)	$J_2'(ka)=0$	2
6	3.8317(3.8364)	$J_0'(ka)=0$	1

Note that data in parenthesis are obtained by dual BEM.

In comparing with the numerical results in Fig.1, Fig.2 shows the analytical results for the absolute values of the determinant in Eqs.(35) $\sim$ (42) by using the circulant property of finite-dimensional space. The spectral properties for the eight matrices can be easily found. The positions of k with zeros of the determinants in Fig.2 for the interior matrices show the eigenvalues for the interior problem, while those for the exterior matrices indicate the fictitious wave number. Good agreement was made for the locations of poles or eigenvalues between the analytical study using circulants (Fig.2) and numerical experiments using the dual BEM program (Fig.1). Only a scale difference can be found. When the number N in circulants approaches to infinity, the continuous system can be simulated analytically.

Concluding remarks

The mechanism of fictitious frequencies in the direct dual BEM has been examined by considering the circulants for the inflence matrices of a circular problem. The discrete system with $2N$ elements can be extended to be infinite for continuous system and analytical solution can be derived. It is found that the irregular values depend on the integral formulation, UT or LM equation used in the dual integral equations, instead of the type of specified boundary condition. Two examples have been given to illustrate this conclusion. Both examples show that the first UT equation results in fictitious frequencies which are associated with the interior acoustic frequency with essential homogeneous boundary conditions, while the second LM equation produces fictitious frequencies which are associated with the interior eigenfrequency with natural homogeneous boundary conditions. The numerical results using the dual BEM program agree very well with the analytical solutions using circulants.

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Fig.1 The absolute values of the determinant versus *k* for the eight influence matrices by using the dual BEM program (No. of boundary elements = 20).

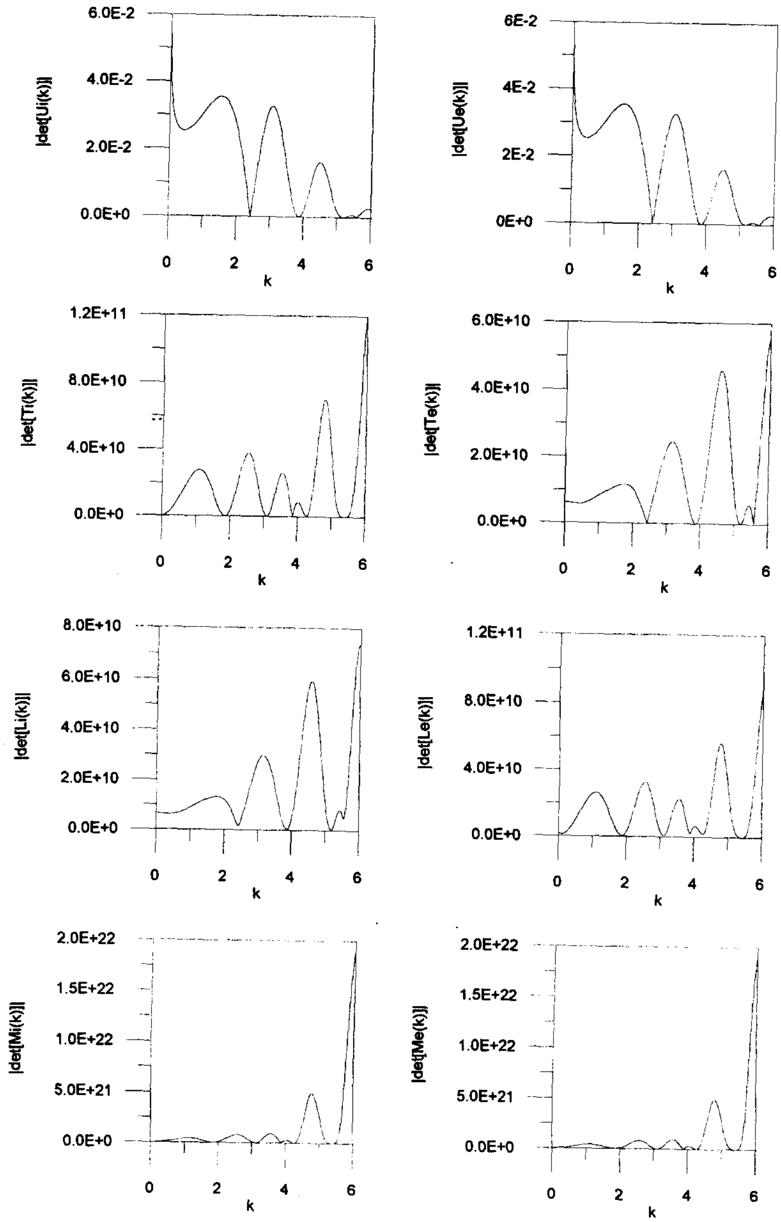

Fig.2 The absolute values of the determinant versus *k* for the eight influence matrices by using circulants (N=10).