邊界積分求積法於等向性梁構件之Saint-Venant扭轉問題的數值模擬分析

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摘 要

本文針對等向性梁構件之Saint-Venant扭轉問題，採用邊界積分求積法（Boundary Integral Quadrature Method, BIQM）進行數值模擬，並探討以Prandtl應力函數與翹曲函數做為欲解場量的適用性與收斂效率。在邊界積分求積法中利用自適性正確解引入至邊界積分方程式上，不僅可以技巧性地計算的奇異積分，也可以省去邊界固體角計算。而在邊界積分方程式上使用高斯求積法與利用參數表示法邊界曲線後，積分方程式便可直接轉換成線性代數方程式，由於過程無須建立元素網格，因此邊界積分求積法實屬一種無網格法。本文考慮兩種斷面幾何進行驗證，包括T130梁斷面以及箱型橋梁斷面，藉由配置不同數量之高斯積分點，以分析複雜斷面於扭轉作用下之力學行為。從數值結果顯示，隨著高斯積分點數的增加，利用邊界積分求積法所計算之扭轉常數逐漸收斂，且與既有文獻中的數值結果高度一致，顯示本法具有良好的數值準確性。綜合而言，本文所提出之邊界積分求積法應用於扭轉問題分析時，不僅具備邊界型方法降低維度之優勢，且也無須邊界元素網格的建立，同時亦能有效處理複雜斷面幾何，顯示其於結構工程扭轉分析與橋梁構件設計上具有良好之應用潛力。

This paper addresses the Saint-Venant torsion problem of an isotropic beam by employing the boundary integral quadrature method (BIQM). It investigates the applicability and convergence efficiency of using the Prandtl stress function and the warping function as the field variables. In the BIQM, by introducing the adaptive exact solution into the boundary integral equation, the singular integral can be sidestepped and the calculation of the solid angle can be bypassed. Furthermore, by adopting Gauss quadrature for the boundary integral and by utilizing a parametric representation for the boundary curves, the boundary integral equation can be directly transformed into a linear algebraic equation. Since the process does not require the construction of element meshes, the BIQM is essentially a meshless method. Two types of cross-sections are considered to verify, including a T130 beam section and a box-girder bridge section. By allocating different numbers of Gaussian points, the mechanical behavior of complex sections under torsional action is analyzed. Numerical results demonstrate that as the number of Gaussian points increases, the torsional constant gradually converges. It shows high consistency with the numerical results in the literature, indicating that the BIQM possesses excellent numerical accuracy. In conclusion, when the BIQM proposed in this paper is applied to torsion problems, it not only has the advantage of dimensionality reduction but also eliminates the need for model preparation of boundary element meshes. Simultaneously, it can effectively deal with complex sectional geometries and demonstrate excellent application potential in the torsion analysis of structural engineering and the design of bridge components.

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