A NEW METHOD FOR LAPLACE'S EQUATION IN TWO-DIMENSIONAL REGIONS WITH CIRCULAR HOLES

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Outlines

- Motivation
- Boundary integral equations
- Degenerate kernels
- Present method
- Numerical examples
- Conclusions

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Motivation

Laplace's problems with circular holes:

- Steady state heat conduction of tube
- Flow of incompressible flow around cylinders
- Electrostatic fields of wires
- Torsion bar with circular holes

- Caulk et al. (1983)-steady heat conduction with circular holes
- Bird and Steele (1992)- harmonic and biharmonic problems with circular holes
- Mogilevskaya et al. (2002)- elasticity problems with circular boundaries
- Ling (1943)- torsion of a circular tube

However, they didn't employ the null-field integral equation and degenerate kernels to fully capture the circular boundary, although they all employed Fourier series expansion.

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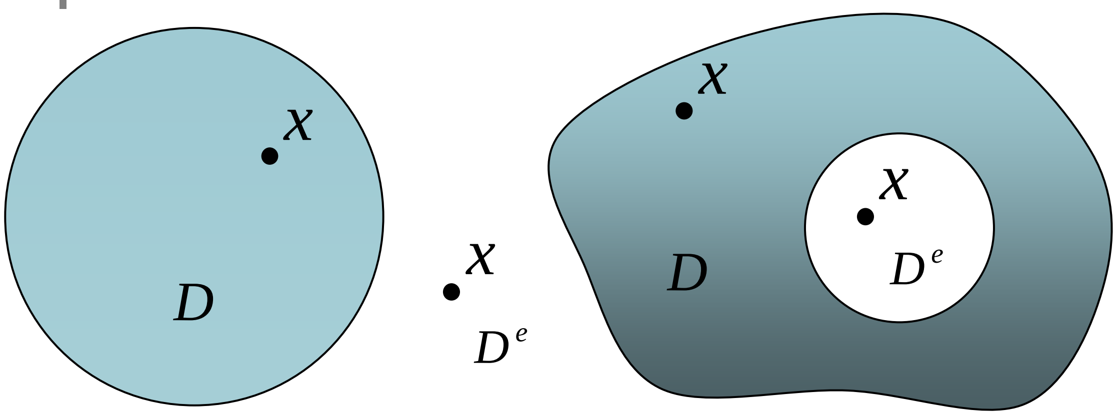
Boundary integral equations



where

$$U(s,x) = \ln|x-s| = \ln r$$

$$T(s,x) = \frac{\partial U(s,x)}{\partial n_s}$$

$$t(s) = \frac{\partial u(s)}{\partial n_s}$$

$$2\pi u(x) = \int_B T(s,x)u(s)dB(s) - \int_B U(s,x)t(s)dB(s), \; x \in D$$

$$0 = \int_B T(s,x)u(s)dB(s) - \int_B U(s,x)t(s)dB(s), \; x \in D^e$$

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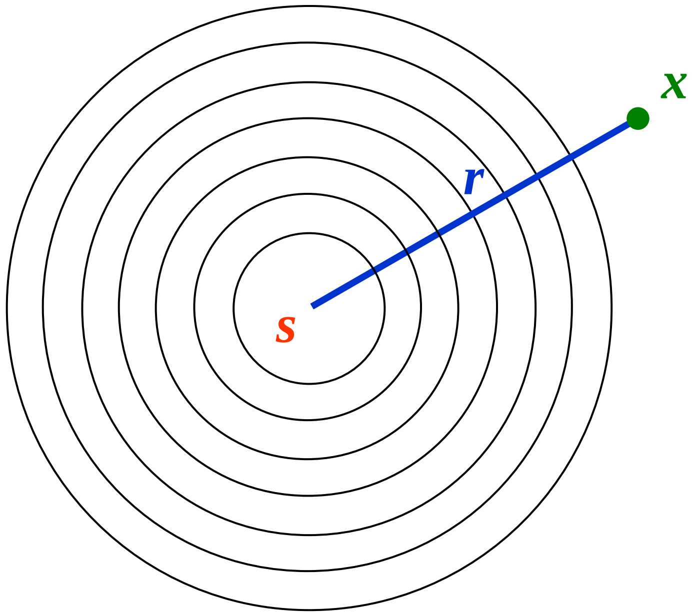
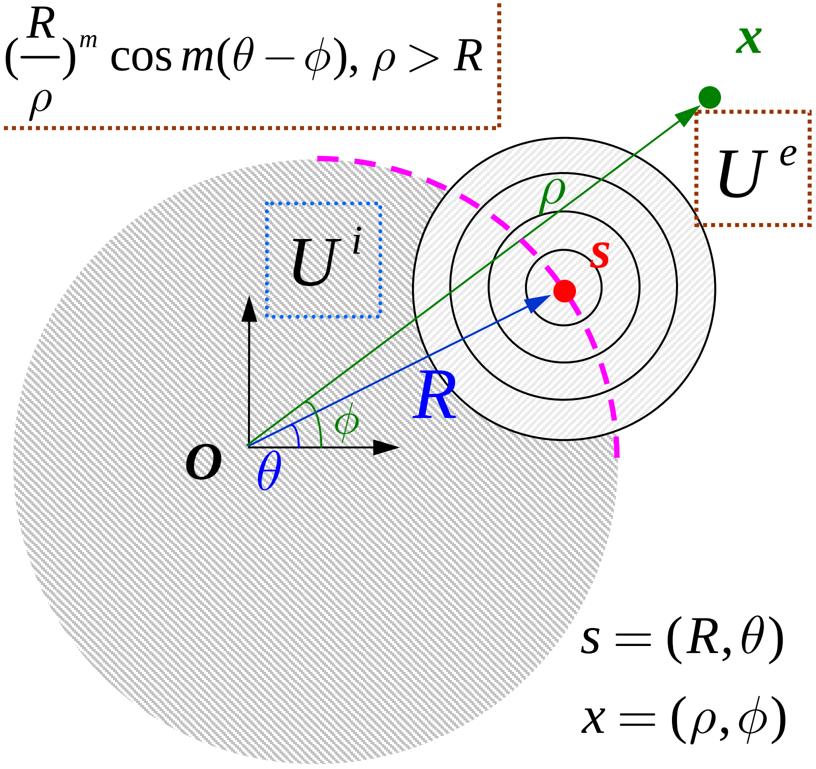
Degenerate kernels

$$U(s,x)=\ln r=\begin{cases} U^{i}(s,x)=\ln R-\sum_{m=1}^{\infty}\frac{1}{m}\left(\frac{\rho}{R}\right)^{m}\cos m(\theta-\phi), & R>\rho \\ U^{e}(s,x)=\ln \rho-\sum_{m=1}^{\infty}\frac{1}{m}\left(\frac{R}{\rho}\right)^{m}\cos m(\theta-\phi), & \rho>R \end{cases}$$





$s=(R,\theta)$

$x=(\rho,\phi)$

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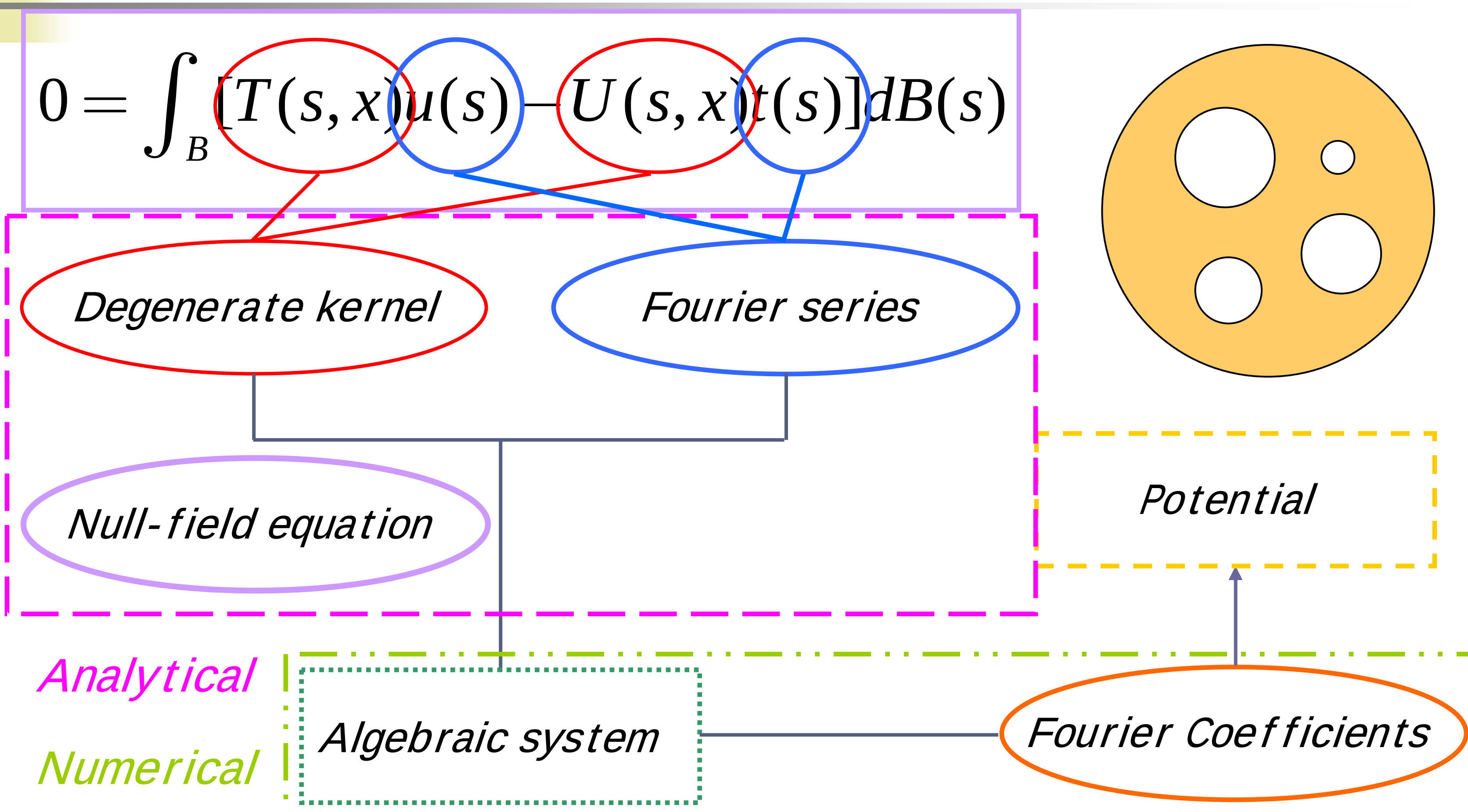
Flowchart of present method

$$0 = \int_B [T(s,x)u(s) - U(s,x)t(s)]dB(s)$$

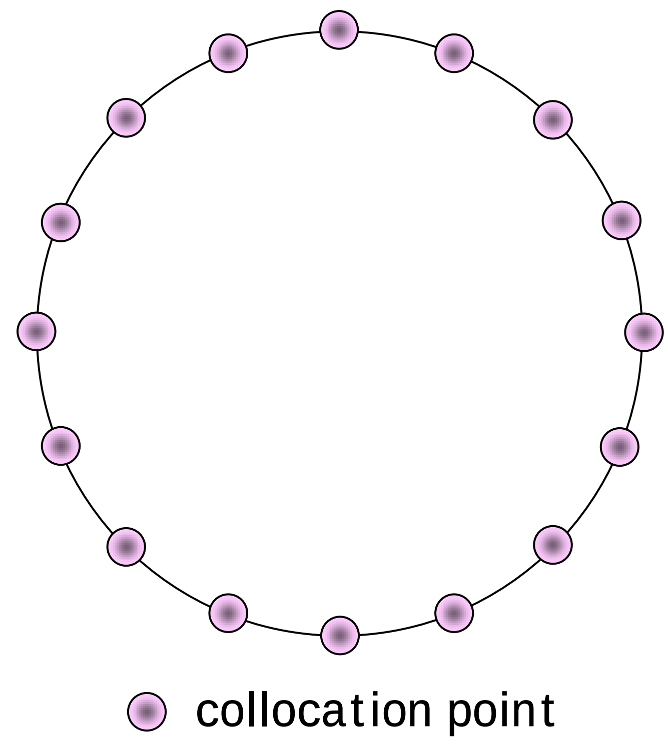


Collocation points

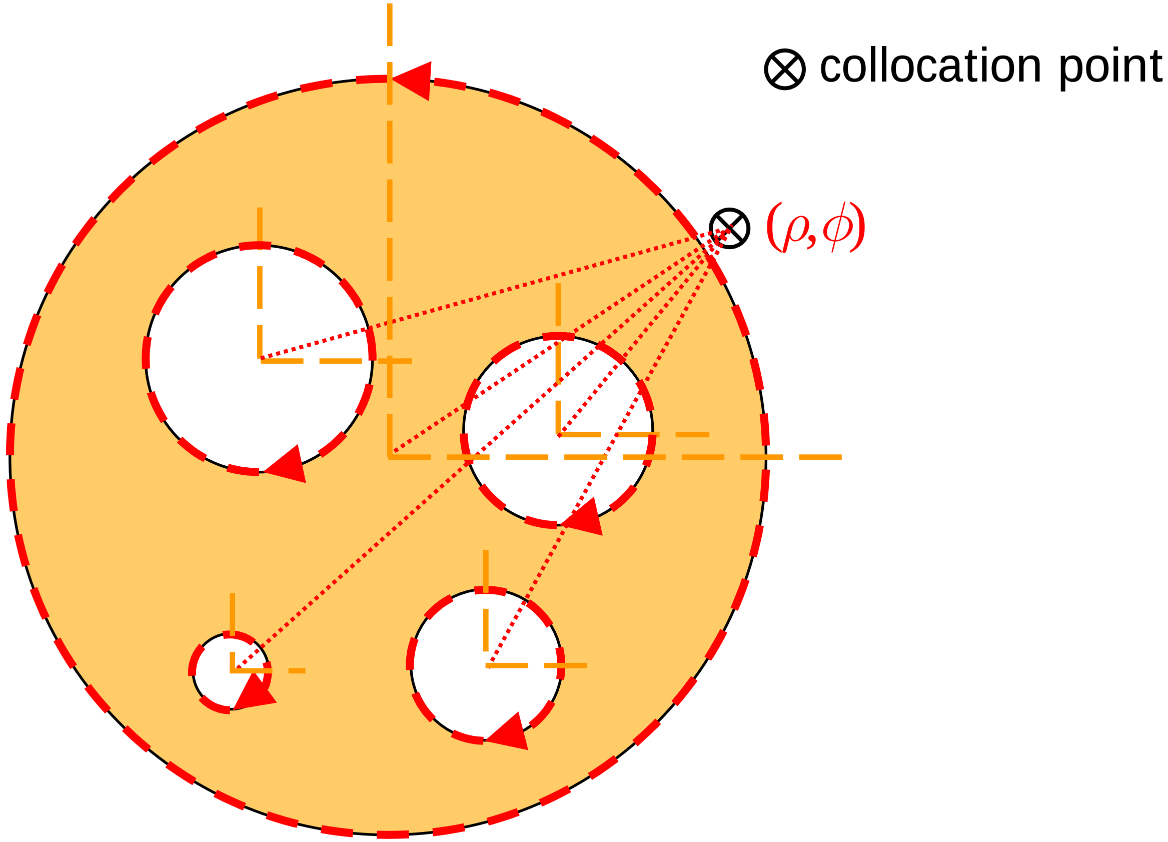
$$u(s) = a_0 + \sum_{n=1}^{M} (a_n \cos n\theta + b_n \sin n\theta)$$

$$t(s) = p_0 + \sum_{n=1}^{M} (p_n \cos n\theta + q_n \sin n\theta)$$

2M+1 unknown Fourier coefficients

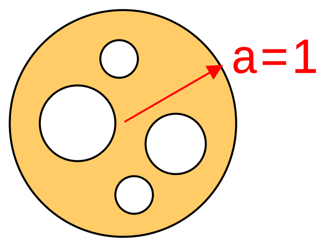


The idea of the present formulation



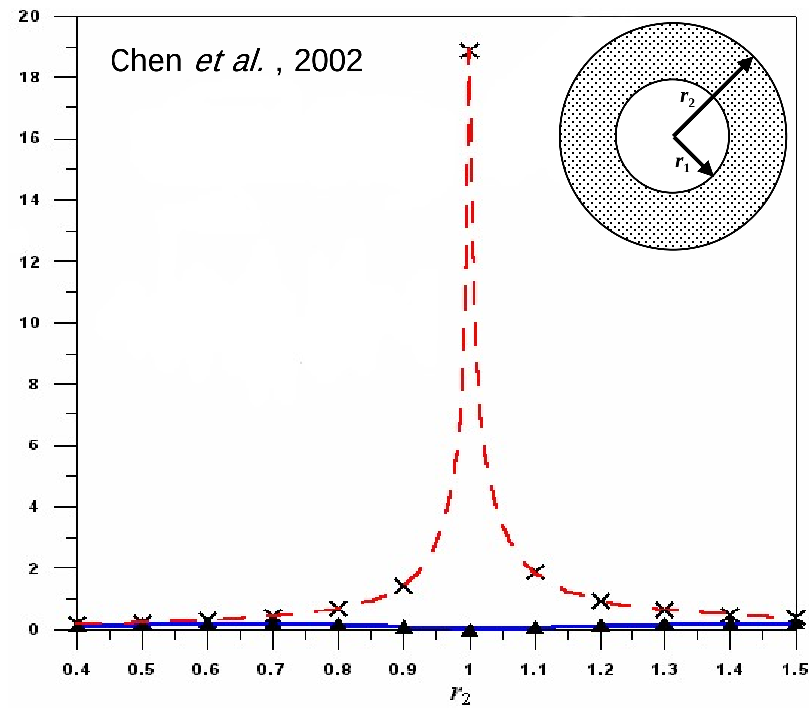
Discussions on degenerate scale

Linear algebraic system



Singular

$$\begin{bmatrix} \cdots & 2\pi a\ln a & \cdots \\ \vdots & \vdots & \vdots \\ \cdots & 2\pi a \ln a & \cdots \end{bmatrix} \begin{Bmatrix} \vdots \\ p_{oj} \\ p_{mj} \\ q_{mj} \\ \vdots \end{Bmatrix} = [T] \begin{Bmatrix} \vdots \\ a_{oj} \\ a_{mj} \\ b_{mj} \\ \vdots \end{Bmatrix}$$



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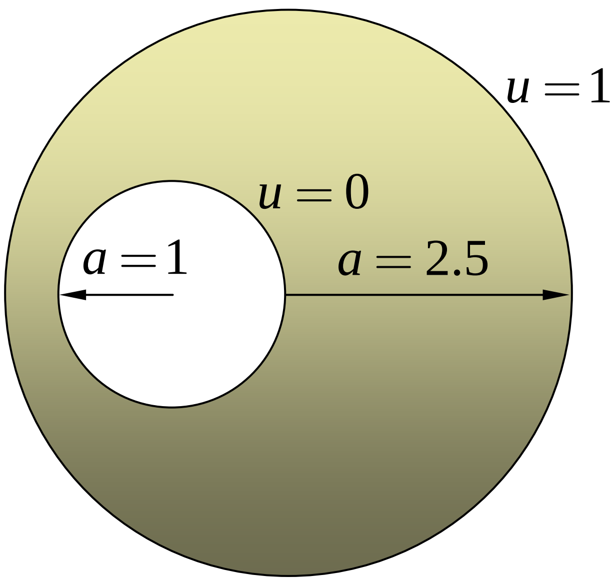
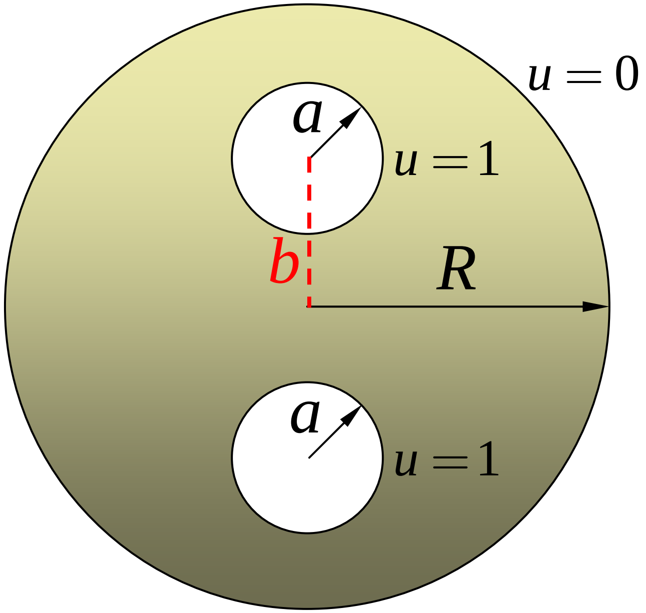
Numerical examples

Case 1



Case 2

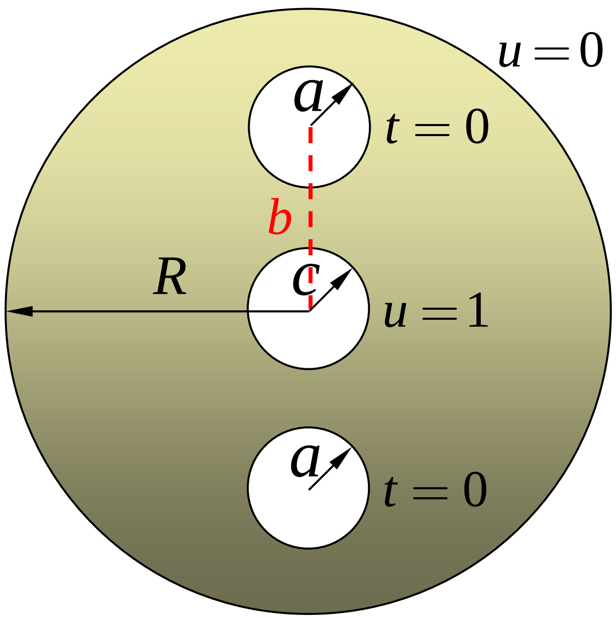
a=0.5, b=1.0, R=2.0



Numerical examples

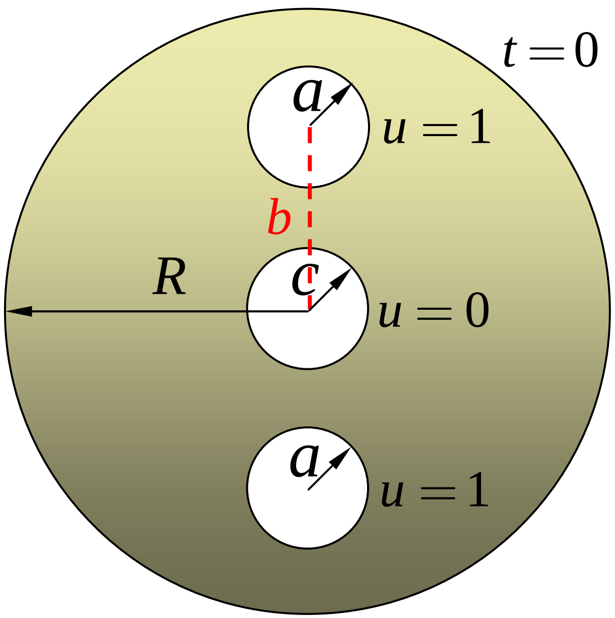
Case 3

a=c=0.4, b=1.2, R=2.0



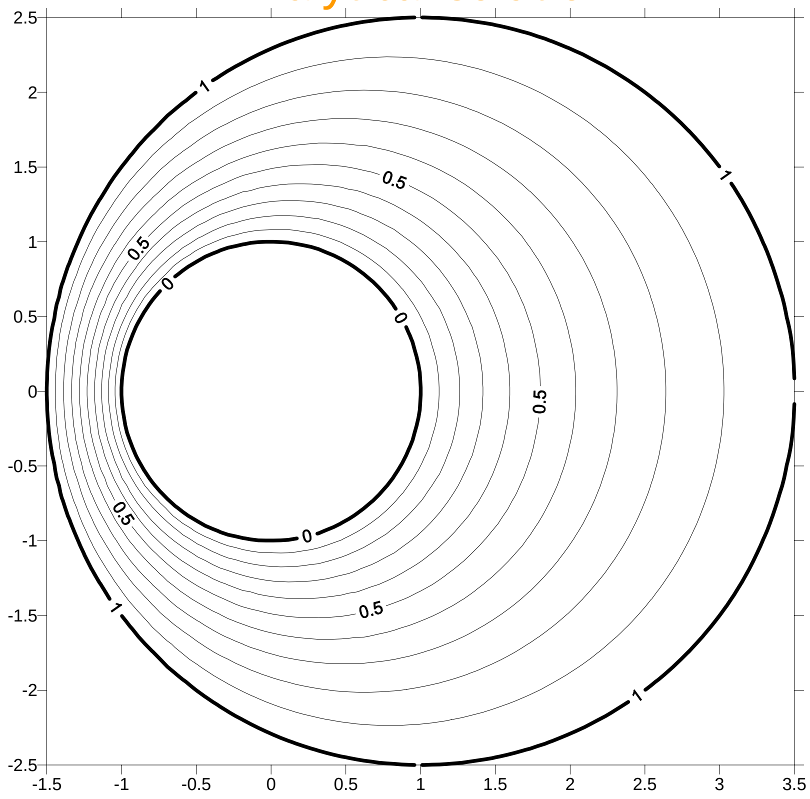
Case 4

a=c=0.4, b=1.2, R=2.0

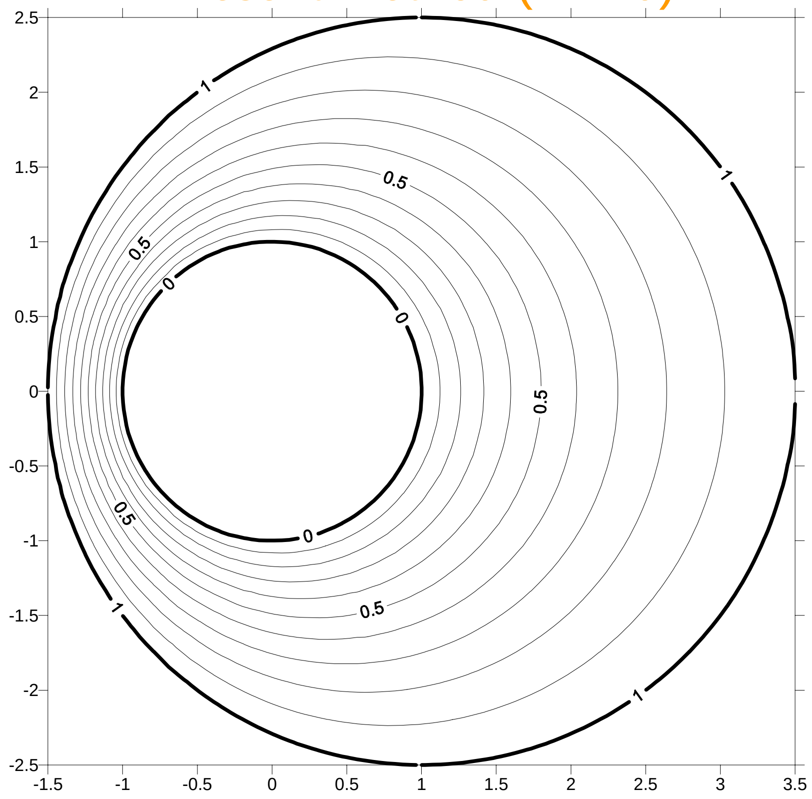


Contour of potential (case 1)

Analytical solution

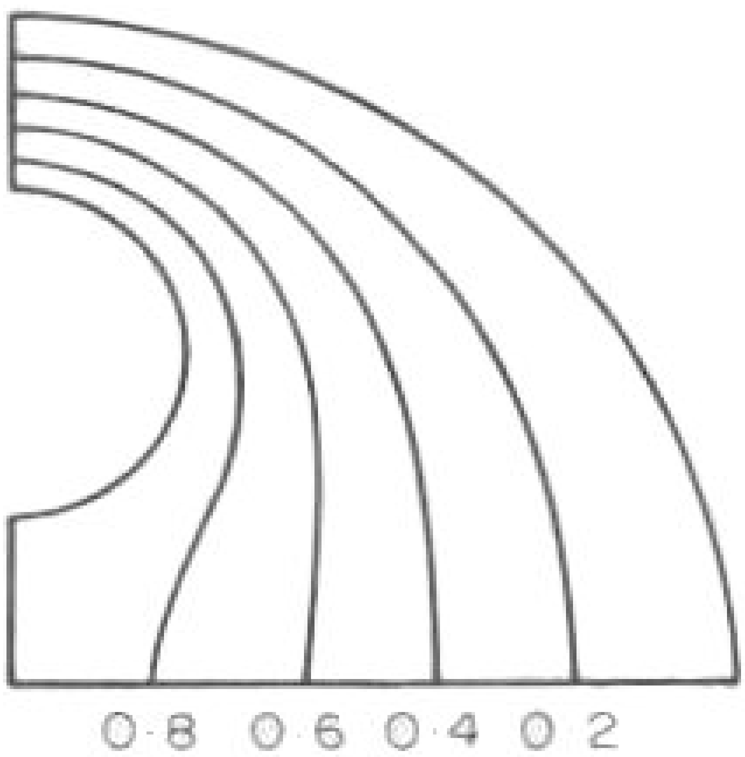


Present method (M=10)



Quarter part of contour of potential (case 2)

Caulk (1986)



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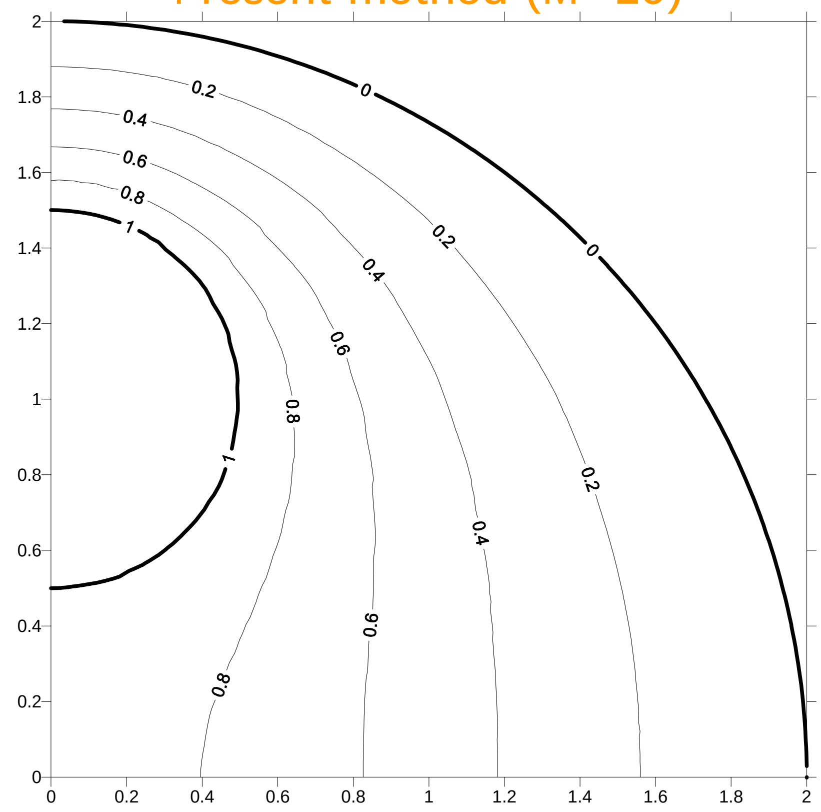

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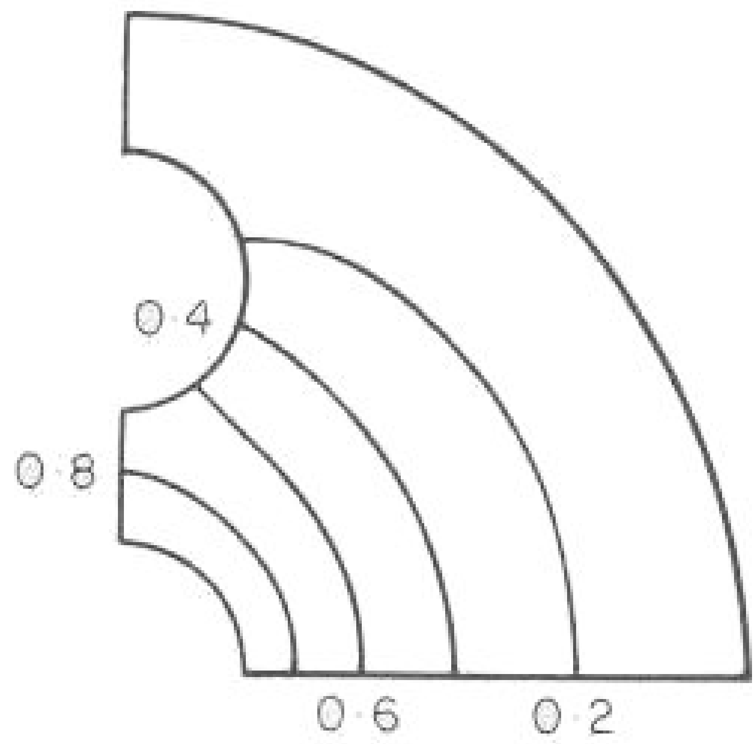

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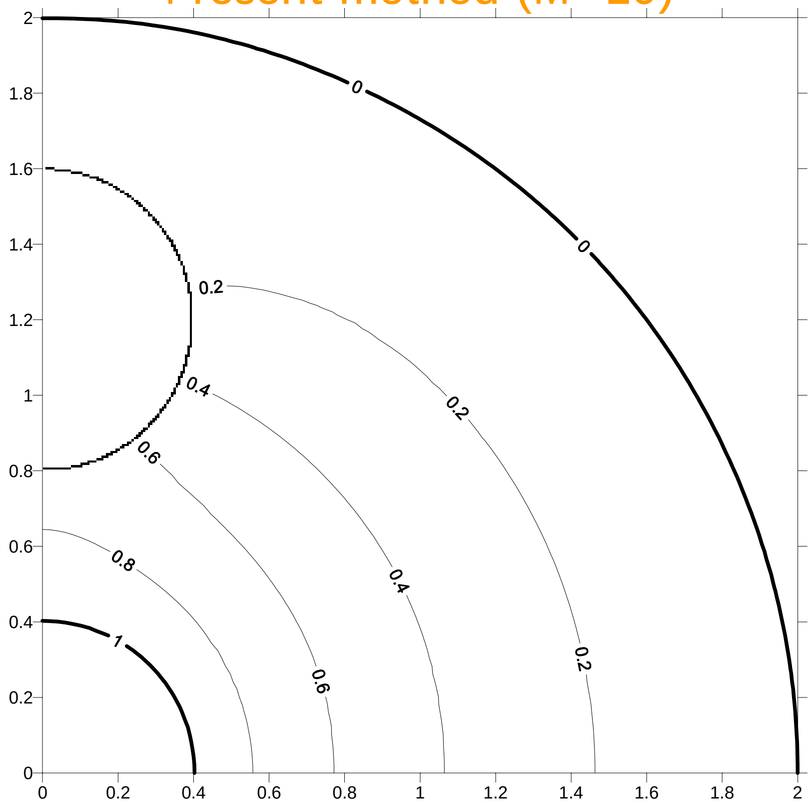
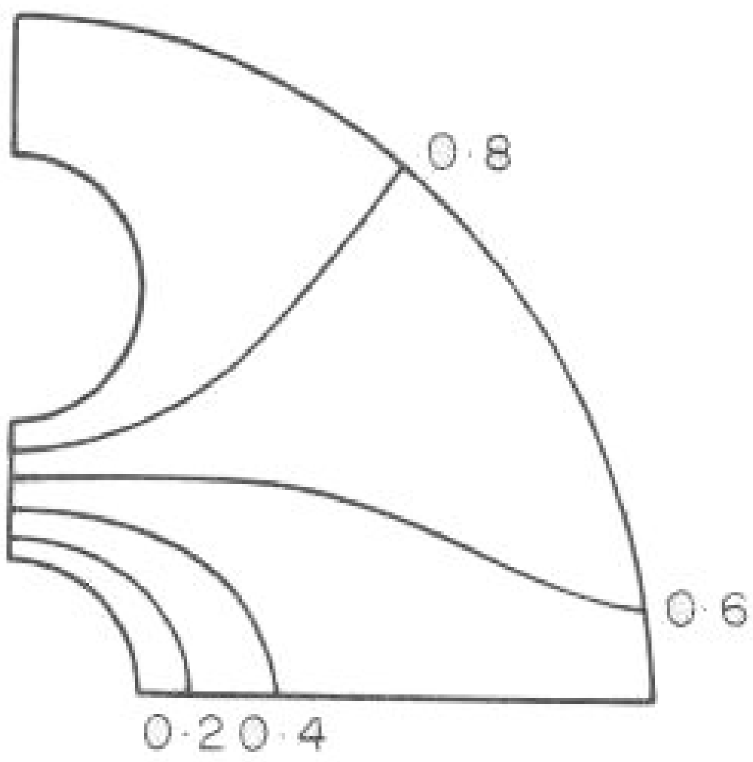


Quarter part of contour of potential (case 4)

Caulk (1986)



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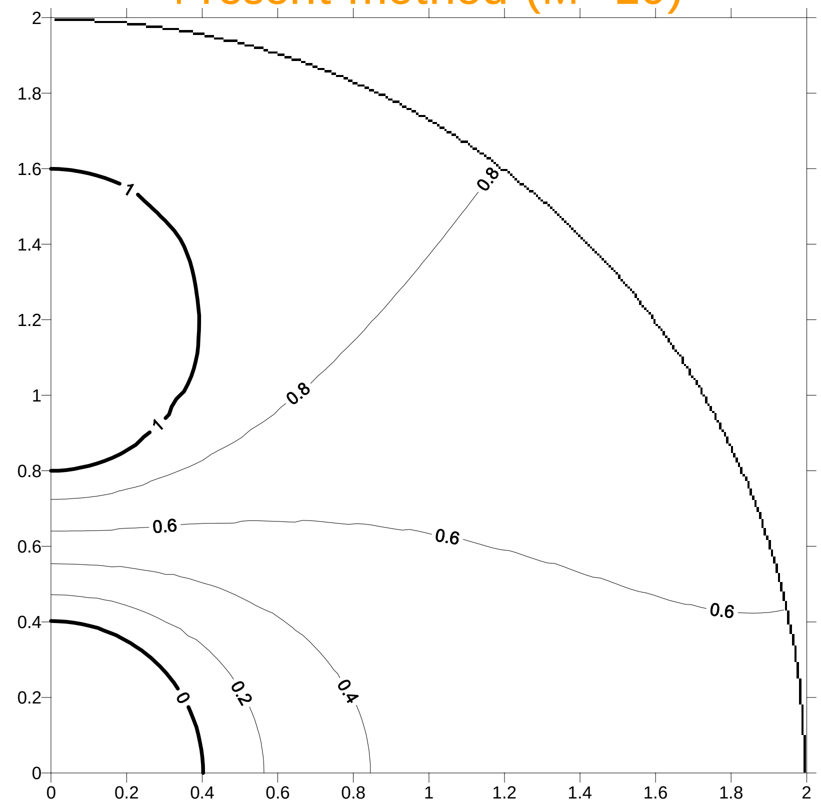

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Conclusions

- A novel method using degenerate kernels, Fourier series and null-field integral equation has been successfully proposed to solve problems with circular boundaries.
- The method shows great generality and versatility for problems with multiple circular holes of arbitrary radii and position.
- Numerical results agree well with available exact solutions and Caulk's data for only few terms of Fourier series.

The end

Thanks for your attentions.

Your comment is much appreciated.

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http://msvlab.hre.ntou.edu.tw