Relationship between the Green's matrix of SVD and the Green's function matrix of SVE for exterior acoustics

外域聲場中格林函數矩陣的SVD與SVE的關聯

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Outlines

- Motivation
- Green's function
- SVD (Singular value decomposition)
- SVE (Singular value expansion)
- Relationship between the SVD and the SVE
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Motivation

- Modal representation for the exterior acoustic field becomes important.
- What are the physical meanings of the SVD in exterior acoustic problem.
- What are the mathematical mechanism of the unitary matrices and singular value.

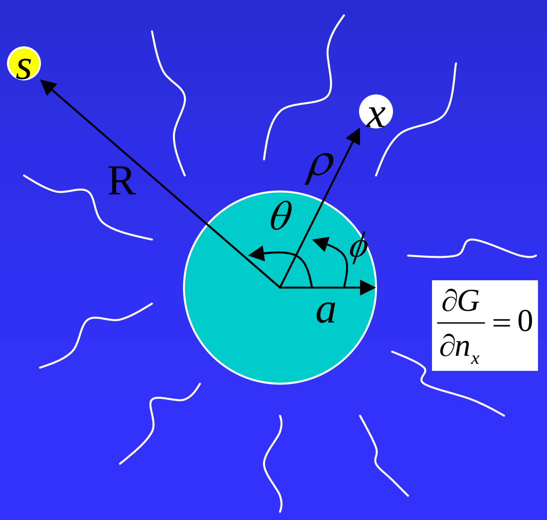
What is the Green’s function

The Green’s function relating to the acoustic pressure of field to the strengths of source on the boundary

$$G(s,x)=\frac{-i\pi}{2}\sum_{n=-\infty}^{\infty}\frac{H_n^{(1)}{}'(ka)J_n(kR)-H_n^{(1)}(kR)J_n'(ka)}{H_n^{(1)}{}'(ka)}H_n^{(1)}(k\rho)\Theta_n(\bar{\phi})\Theta_n^{+}(\theta)$$

1. Degenerate kernel
2. Image method

$$(\nabla^2+k^2)G(x,s)=2\pi\delta(x-s)$$



- **Source points** (yellow)
- **Field points** (white)

Degenerate kernels

$$(\nabla^2 + k^2)U(x,s) = 2\pi\delta(x-s)$$

$$U(s,x) = \frac{-i\pi H_0^{(1)}(kr)}{2}$$ **(closed-form),** $r = |s-x|$

$$U(x,s) = \begin{cases} U^i(\rho,\bar{\phi};R,\theta) = \sum_{n=-\infty}^{\infty} \frac{-i\pi}{2} H_n^{(1)}(k\rho) J_n(kR)\Theta_n(\theta)\Theta_n^{+}(\bar{\phi}), \ \rho > R, \\ U^e(\rho,\bar{\phi};R,\theta) = \sum_{n=-\infty}^{\infty} \frac{-i\pi}{2} H_n^{(1)}(kR) J_n(k\rho)\Theta_n(\theta)\Theta_n^{+}(\bar{\phi}), \ \rho < R, \end{cases}$$

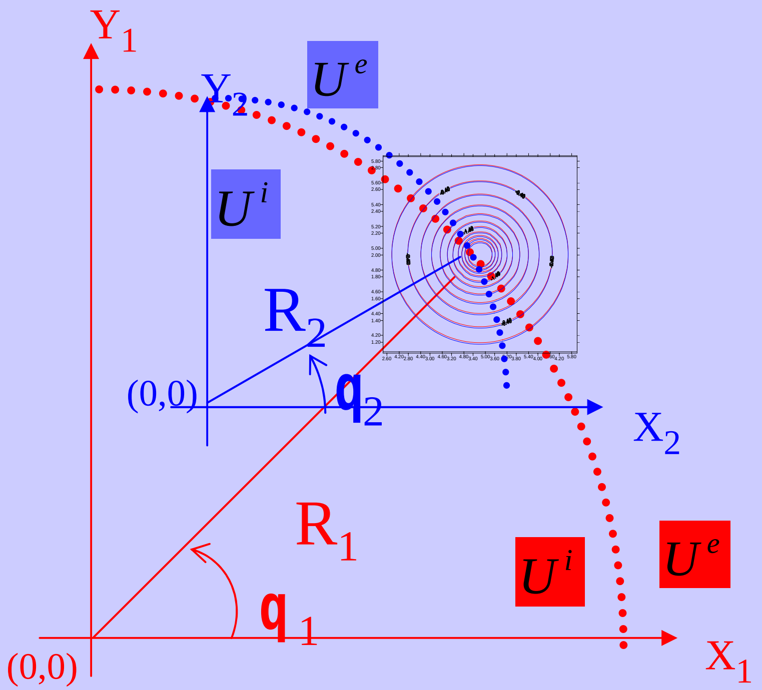
$H_0^{(1)}(kr)$:The first kind Hankel function with order *0*.

$s = (R,\theta)$: source point

$x = (\rho,\bar{\phi})$: field point

$$\Theta_n(\theta) = e^{in\theta}$$

$$\Theta_n(\bar{\phi}) = e^{in\bar{\phi}}$$



The image method of acoustic field

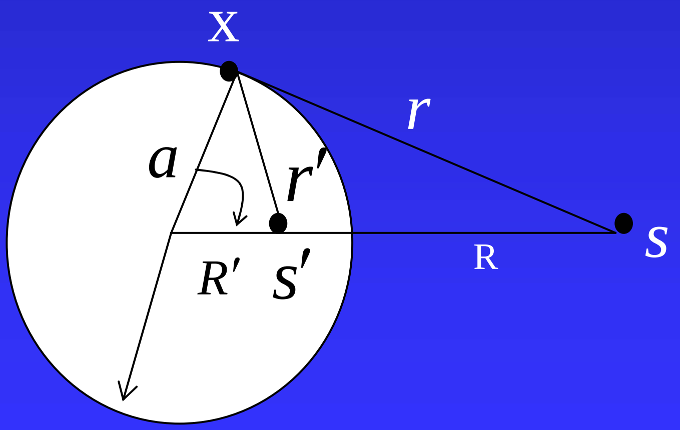
$$G.E. \quad (\nabla^2 + k^2) G(x,s) = 2\pi\delta(x-s)$$

The Green's function satisfies the B.C.

$$\frac{\partial G}{\partial n_x} = 0, \quad (x \text{ on } B)$$

$$G(x,s) = U^e(\rho, \bar{\phi}; R, \theta) - U^i(\rho, \bar{\phi}; R', \theta)$$

$$J_n(kR') = \frac{H_n^{(1)}(kR)}{H_n^{(1)'}(ka)} J'_n(ka)$$



The Green's matrix

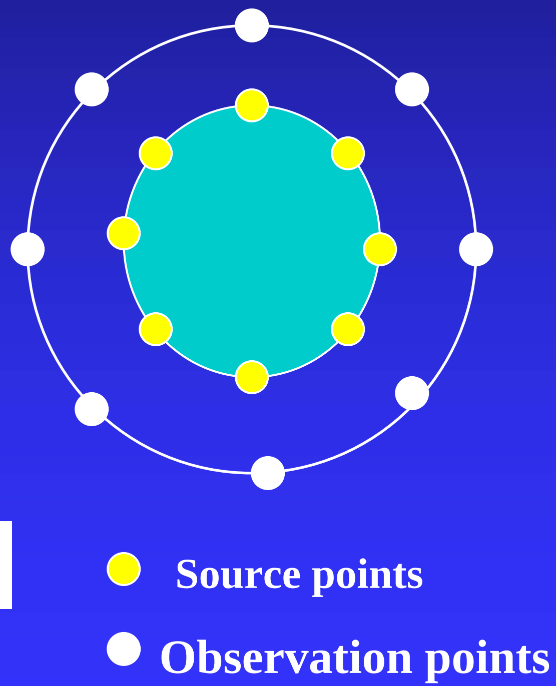
Boundary integral equation (singular integral equation)

$$\pi u(x) = CPV \int_B T(s,x)u(s)dB(s) - RPV \int_B U(s,x)t(s)dB(s), \quad x \in B$$

$$2\pi u(x) = \int_B T(s,x)u(s)dB(s) - \int_B U(s,x)t(s)dB(s), \quad x \in D.$$

CPV : Cauchy principal value

RPV : Reimann principal value



Discrete the boundary integral equation

$$[T_B]\{u_B\} = [U_B]\{t_B\}$$

$$\{u(x)\} = [T_D]\{u_B\} - [U_D]\{t_B\}$$

$$\{u(x)\} = ([T_D][T_B]^{-1}[U_B] - [U_D])\{t_B\} = [G]\{t_B\}$$

G: Green's matrix

Singular value decomposition (SVD)

$$[G]_{P\times V} = \Phi_{P\times P}\Sigma_{P\times V}\Psi^{+}_{V\times V}$$

P: number of the field points

V: number of the source points

+: transpose conjugate

$\Phi^{+}\Phi = \Phi\Phi^{+} = I$ → Left unitary matrix

$\Psi^{+}\Psi = \Psi\Psi^{+} = I$ → Right unitary matrix

$$\Sigma = \begin{bmatrix} \sigma_V & 0 & \cdots & 0 \\ 0 & \sigma_{V-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_1 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix}_{(V+2)\times V}$$

Singular value expansion

$$G(s,x)=\frac{-i\pi}{2}\sum_{n=-\infty}^{\infty}\frac{H_n^{(1)}{}'(ka)J_n(k\rho)-H_n^{(1)}(ka)J_n'(k\rho)}{H_n^{(1)}{}'(ka)}H_n^{(1)}(k\rho)\Theta_n(\bar{\phi})\Theta_n^{+}(\theta)$$

$$[G(s,x)]=\lim_{M\to\infty}\begin{bmatrix}\sum_{n=-M}^{M}g_n\Theta_n(\bar{\phi}_1)\Theta_n^{+}(\theta_1) & \cdots & \sum_{n=-M}^{M}g_n\Theta_n(\bar{\phi}_1)\Theta_n^{+}(\theta_V)\\ \sum_{n=-M}^{M}g_n\Theta_n(\bar{\phi}_2)\Theta_n^{+}(\theta_1) & \cdots & \sum_{n=-M}^{M}g_n\Theta_n(\bar{\phi}_2)\Theta_n^{+}(\theta_V)\\ \vdots & \ddots & \vdots\\ \sum_{n=-M}^{M}g_n\Theta_n(\bar{\phi}_P)\Theta_n^{+}(\theta_1) & \cdots & \sum_{n=-M}^{M}g_n\Theta_n(\bar{\phi}_P)\Theta_n^{+}(\theta_{V1})\end{bmatrix}_{P\times V}$$

$$g_n=i\rho_0 c\frac{H_n^{(1)}(k\rho)}{H_n^{(1)'}(ka)},$$

Singular value expansion

$$[G(s,x)] = \Theta(\bar{\phi}_P)\Lambda\Theta^+(\theta_V)$$

$$= \Gamma(\bar{\phi}_P)\Sigma\Gamma^+(\theta_V)\Theta^+(\theta_V)$$

$$= \Theta(\bar{\phi}_P)\begin{bmatrix} e^{i\varphi_1} & 0 & \cdots & 0 \\ 0 & e^{i\varphi_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{i\varphi_P} \end{bmatrix}_{P\times P} \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & \sigma_V \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}_{P\times V} \begin{bmatrix} e^{i\vartheta_1} & 0 & \cdots & 0 \\ 0 & e^{i\vartheta_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{i\vartheta_V} \end{bmatrix}_{V\times V} \Theta^+(\theta_V)$$

$$[G] = \Phi\Sigma\Psi^+$$

$$\varphi_N = Arg[H_n^{(1)}(k\rho)]$$

$$\vartheta_N = Arg[H_n^{(1)\prime}(ka)]$$

$$\sigma_N = \left| i\rho_0 c \frac{H_n^{(1)}(k\rho)}{H_n^{(1)\prime}(ka)} \right|$$

Relationship between SVD and SVE

$$\Gamma(\bar{\phi}_P) \qquad \Gamma^{+}(\bar{\phi}_P)\Gamma(\bar{\phi}_P) = \frac{I}{P}$$

$$\Phi \longrightarrow \Theta(\bar{\phi}_P)$$

$$\Phi = \Theta(\bar{\phi}_P)\Gamma(\bar{\phi}_P)$$

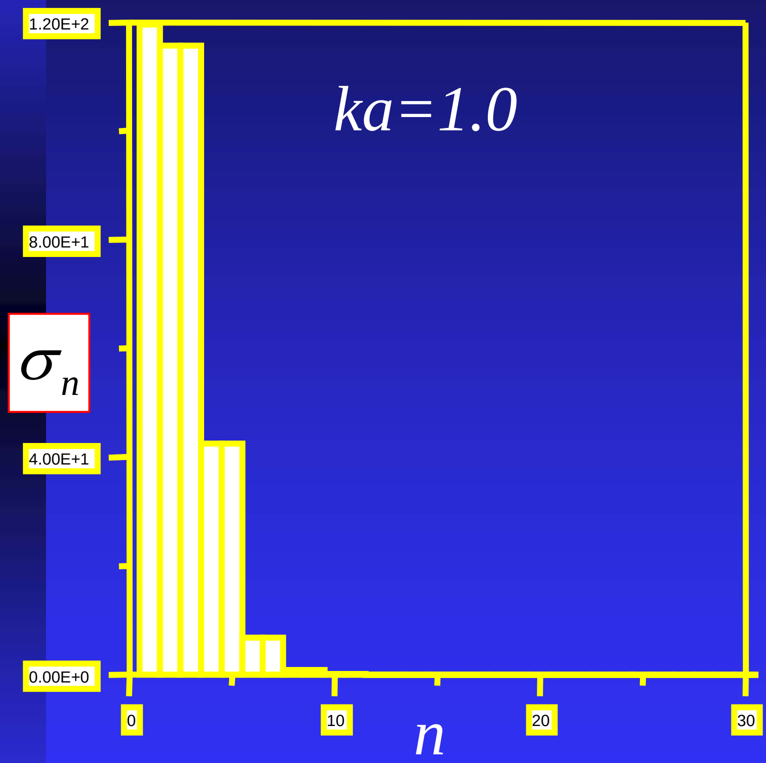
$$\Gamma(\hat{\theta}_V) \qquad \Gamma^{+}(\hat{\theta}_V)\Gamma(\hat{\theta}_V) = \frac{I}{V}$$

$$\Psi \longrightarrow \Theta(\hat{\theta}_V)$$

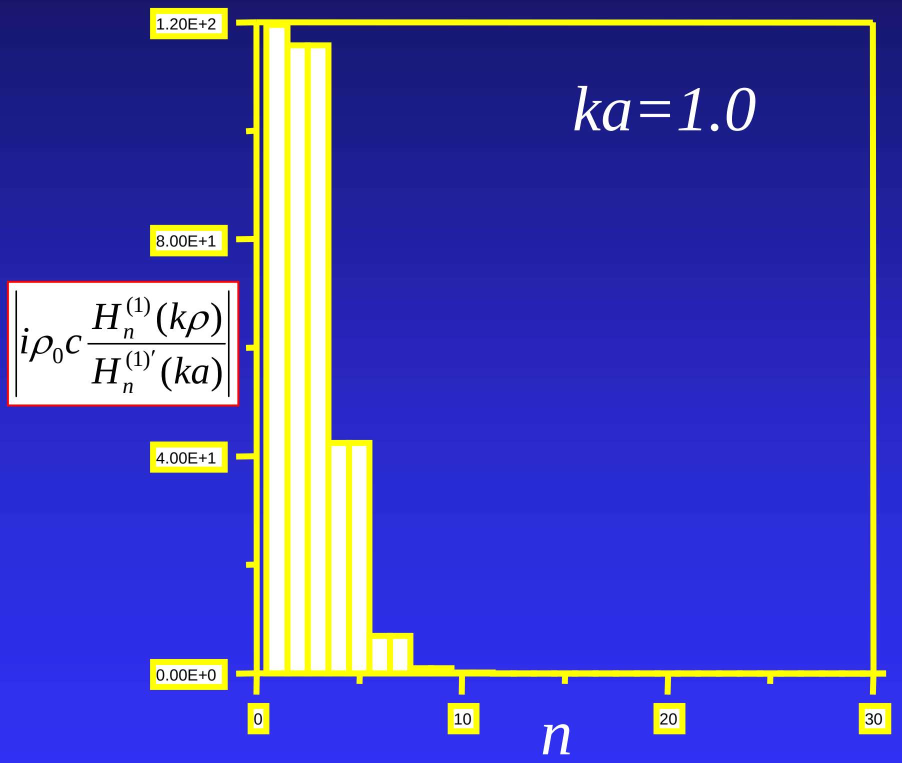
$$\Psi = \Theta(\hat{\theta}_V)\Gamma(\hat{\theta}_V)$$

$$\Sigma = \Gamma^{+}(\bar{\phi}_P)\Lambda\Gamma(\theta_V)$$

Grouping characteristics



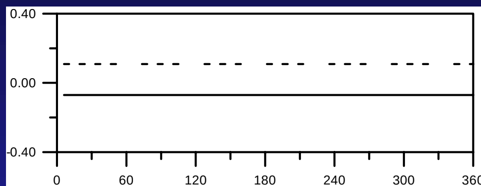
SVD



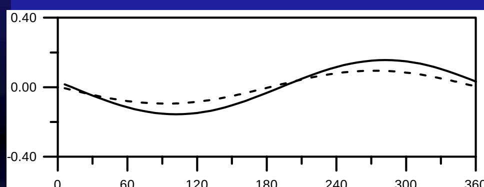
SVE

The left and right singular vectors of SVD

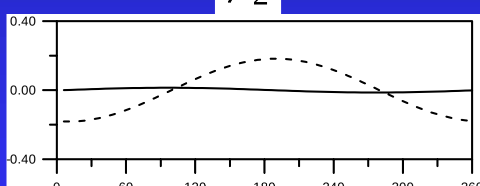
$$[G]_{60\times 60}$$



ϕ_1

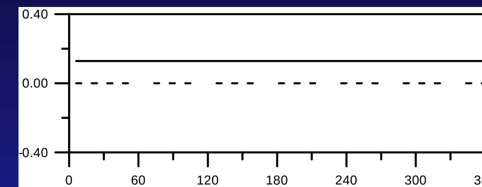


ϕ_2

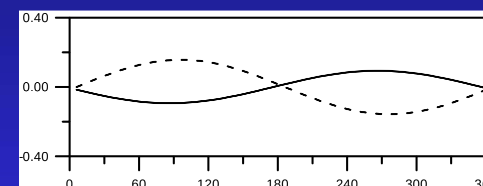


ϕ_3

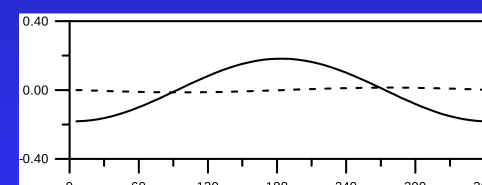
Field mode shapes



φ_1



φ_2



φ_3

Source mode shapes

$$\rho = 10.0\, m,\ ka = 0.01$$

Conclusions

1. The degenerate kernel and image method are employed to derive the Green's function.
2. The physical meaning of the SVD has been examined.
3. The left singular vectors of the SVD of the Green's matrix describing field mode shapes.
4. The right singular vectors of the SVD of the Green's matrix describing source mode shapes.
5. The singular value of the Green's matrix has been obtained and compared with analytical solutions.