第十五屆中國造船暨輪機工程研討會

The method of fundamental solutions for two-dimensional exterior acoustics

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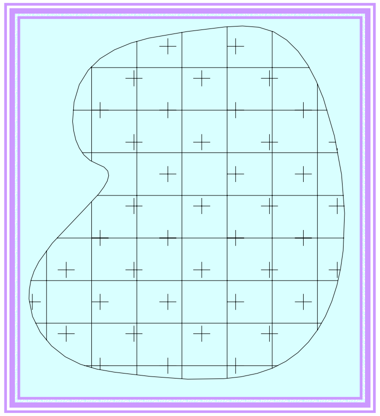
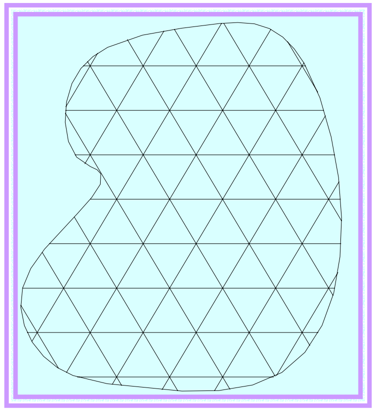
Outlines

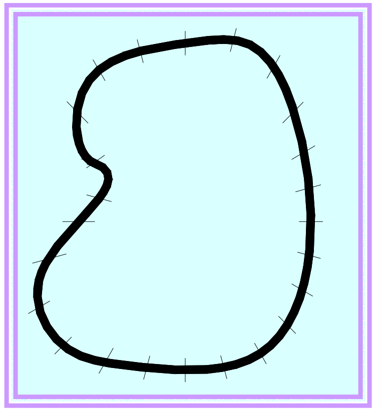
Introduction

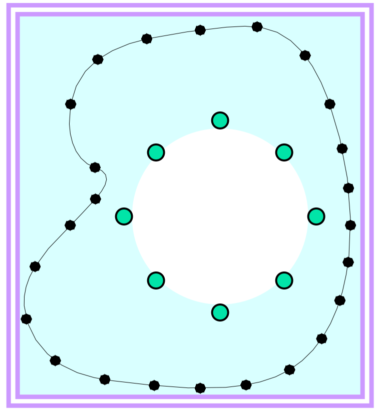
Methods of solution

Finite Difference Method	Finite Element Method	Boundary Element Method	MFS









Finite Difference Method	Finite Element Method	Boundary Element Method	MFS
Mesh ¥	Mesh ¥	Mesh ¥ Singular integral ¥	

Outlines

1. Introduction

2. Method of fundamental solutions

3. Fictitious frequency

4.Conclusions

Radiation of a cylinder

Governing Equation:

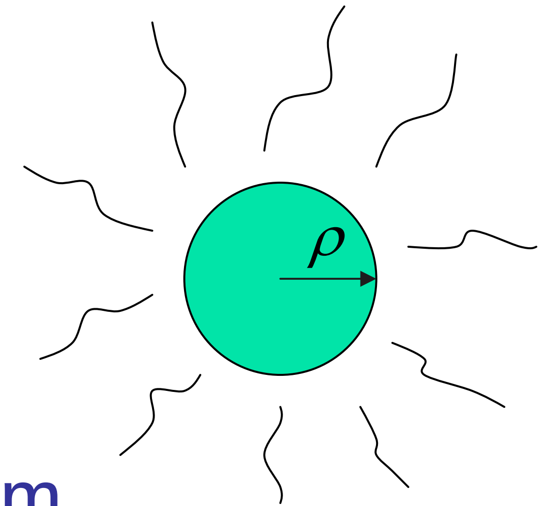
$$(\nabla^2 + k^2)u(x) = 0,\ x \in \Omega$$

∇^2 is the Laplacican operator

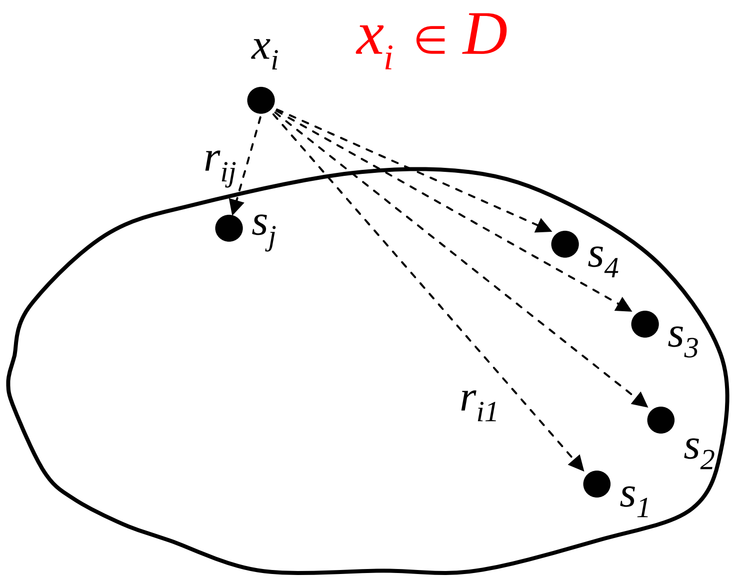
u is the velocity potential

k is the wave number

Ω is the domain of the problem



Field representation using MFS



Field representation

$$u(x_i) = \sum_j A_j U(s_j, x_i)$$

$$U(s,x) = \frac{-i\pi}{2} H_0^{(1)}(kr)$$

$$r \equiv |s - x|$$

$H_0^{(1)}(kr)$ **: Hankel function**

Fundamental solution

Fundamental solution

$$U(s,x) = \frac{-i\pi}{2} H_0^{(1)}(kr)$$

U satisfies

$$\nabla^2 U(s,x) + k^2 U(s,x) = 2\pi\delta(x-s)$$

δ is the Dirac delta function

x is the collocation point

s is the source point

Pressure and velocity

Pressure $$u(x) = \sum_{j=1}^{2N} U(s_j, x) A(s_j)$$

Velocity $$t(x) = \sum_{j=1}^{2N} L(s_j, x) A(s_j)$$

$$t(x) = \frac{\partial u(x)}{\partial n_x}, \qquad L(s,x) = \frac{\partial U(s,x)}{\partial n_s}$$

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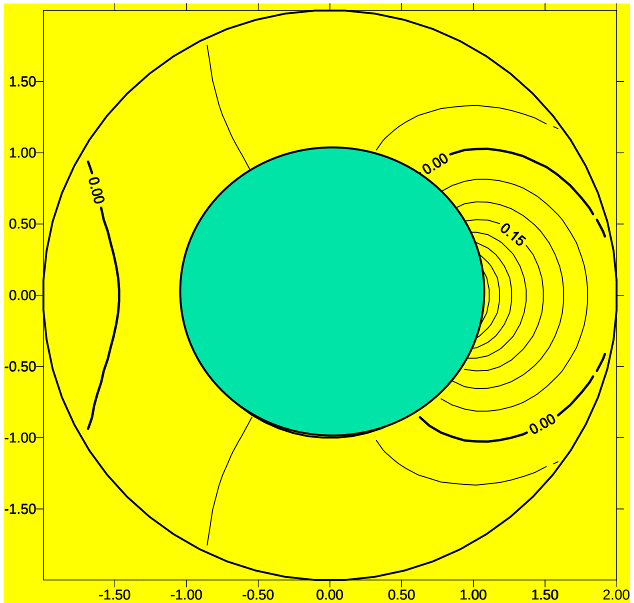
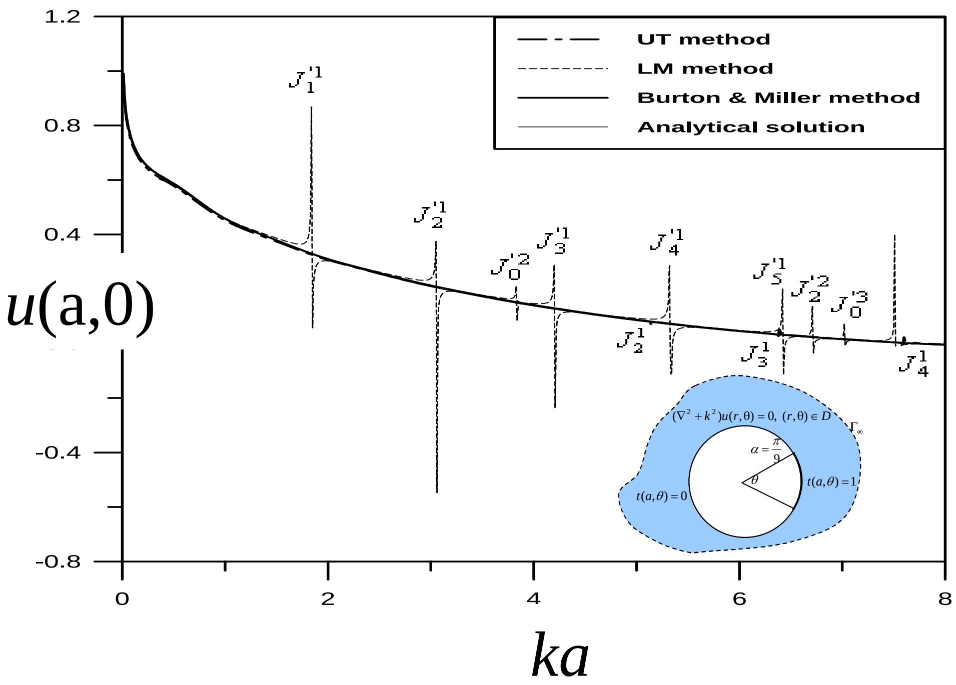
Fictitious frequency (BEM)





Boundary element method

Indirect formulation

Single layer approach

$$u(x) = \int_{B'} U(s,x)\phi(s)\,dB(s) \Rightarrow \{u\} = [U]\{\phi\},$$

$$t(x) = \int_{B'} L(s,x)\phi(s)dB(s) \Rightarrow \{t\} = [L]\{\phi\}.$$

Double layer approach

$$u(x) = \int_{B'} T(s,x)\psi(s)\,dB(s) \Rightarrow \{u\} = [T]\{\psi\},$$

$$t(x) = \int_{B'} M(s,x)\psi(s)dB(s) \Rightarrow \{t\} = [M]\{\psi\}.$$

Method of fundamental solution (MFS)

$$\Leftrightarrow u(x) = \sum_{j=1}^{2N} U(s_j, x)A(s_j)$$

$$\Leftrightarrow t(x) = \sum_{j=1}^{2N} L(s_j, x)A(s_j)$$

Occurrence of fictitious frequency using BEM

	Direct method		Indirect method	
	UT	LM	UL Single layer	TM Double layer
Dirichlet B.C.	$J_n(k\rho)$	$J'_n(k\rho)$	$J_n(k\rho)$	$J'_n(k\rho)$
Neumann B.C.	$J_n(k\rho)$	$J'_n(k\rho)$	$J_n(k\rho)$	$J'_n(k\rho)$

Fictitious frequency problem in MFS

1. Miller R. D., Moyer E. T., Huang H. and Uberall H. A comparison between the boundary element method and the wave superposition approach for the analysis of the scattered field from rigid bodies and elastic shells. JASA 1991; 89; pp.2185-2196.

2. Kondapalli, P. S., D. J. Shippy and G. Fairweather (1992) "Analysis of acoustic scattering in fluids and solids by the method of fundamental solutions," JASA Vol.91, No.4, pp-1844-1854.

Fictitious frequency problem in MFS

However, numerical experiments reported in Miller et al. (1991) and Kondapalli et al. (1992) for the MFS solution of various scattering problems do not indicate the presence of irregular frequencies.

The occurrence of the fictitious frequency

By using the circulants and degenerate kernels, we have the eigenvalue of [*U*] and [*L*]

$$\lambda_l^U = \pi^2 \rho(-iJ_l(k\rho) + Y_l(k\rho))J_l(kR),$$

$$\mu_l^L = \pi^2 k\rho(-iJ_l^{'}(k\rho) + Y_l^{'}(k\rho))J_l(kR),$$

$$l = 0, \pm 1, \pm 2, \cdots \pm (N-1), N$$

The occurrence of the fictitious frequency

For the Dirichlet problem $u = \bar{u}$

$$\{\bar{u}\} = [U]\{A\}$$

$$\{A\} = [U]^{-1}\{\bar{u}\}$$

the possible fictitious frequencies occur at the position where k satisfies

$$H_l^{(1)}(k\rho)J_l(kR) = 0,\ l = 0, \pm 1, \cdots, \pm(N-1), N$$

the k value satisfying the equation implies

$$J_l(kR) = 0.$$

The occurrence of the fictitious frequency

For the Neumann problem $t=\bar{t}$

$$\{\bar{t}\}=[L]\{A\}$$

$$\{A\}=[L]^{-1}\{\bar{t}\}$$

the possible fictitious frequencies occur at the position where k satisfies

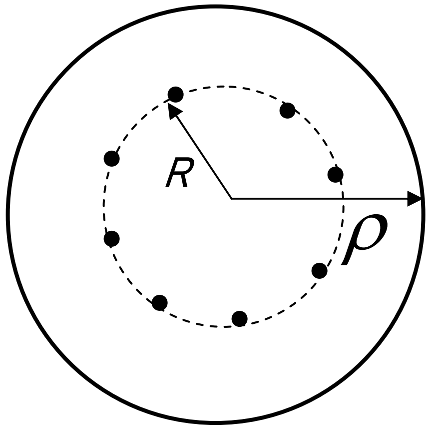
$$H_l'^{(1)}(k\rho)J_l(kR)=0,\ l=0,\pm1,\cdots,\pm(N-1),N$$

the k value satisfying the equation implies

$$J_l(kR)=0.$$

Occurrence of fictitious frequency using MFS

	MFS
Dirichlet B.C.	$J_l(kR)$
Neumann B.C.	$J_l(kR)$



Burton & Miller method

$$u(x_i)=\sum_j\left(U(s_j,x_i)+\frac{i}{k}\frac{\partial U(s_j,x_i)}{\partial n_s}\right)\varphi(s_j)$$

$$t(x_i)=\sum_j\left(\frac{\partial U(s_j,x_i)}{\partial n_x}+\frac{i}{k}\frac{\partial^2 U(s_j,x_i)}{\partial n_x \partial n_s}\right)\varphi(s_j)$$

φ is the density of mixed potential.

Burton & Miller method

For the Dirichlet problem, we have

$$H_l^{(1)}(k\rho)(J_l(kR)+\frac{i}{k}J_l'(kR))\neq 0,\ l=0,\pm 1,\cdots,\pm(N-1),N.$$

for any k.

For the Neumann problem, we have

$$H_l'^{(1)}(k\rho)(J_l(kR)+\frac{i}{k}J_l'(kR))\neq 0,\ l=0,\pm 1,\cdots,\pm(N-1),N.$$

for any *k*.

The Burton & Miller method can overcome the fictitious frequency problem.

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Conclusions

1. By using the degenerate kernels and circulants properties, the mechanism why fictitious frequencies occur in the MFS has been examined.

2. The results indicated that the irregular frequency also appears at the eigenvalue of interior problem where the boundary is connected by the source locations.

3. The Burton & Miller technique was demonstrated to filter out the fictitious frequency analytically.

The End

Thanks for your kind attention