Linkage between unit logarithmic capacity in the theory of complex variables and the degenerate scale in BEM/BIEMM

Shyh-Rong Kuo* and Jeng-Tzong Chen
*srkuo@mail.ntou.edu.tw, Department of Harbor and River Engineering,
National Taiwan Ocean University, Keelung, 20224, Taiwan

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Abstract

It is well known that BEM/BIEM result in degenerate scale for two-dimensional Laplace problem subjected to the Dirichlet boundary condition. In such a case, nontrivial boundary flux exist even the trivial boundary potential is given. It is proved that the unit logarithmic capacity in the Riemann conformal mapping with respect to the unit circle results in a null field for the interior domain. The logarithmic capacity is defined as the coefficient of the linear term in the Riemann conformal mapping. When the logarithmic capacity is equal to one, a trivial interior field can be obtained but exterior field is derived to be nonzero using ln function. Two mapping functions, Riemann conformal mapping for geometry and log function for physics, are both required. This matches well with the BEM result that an interior trivial field has nonzero boundary flux in case of degenerate scale. Regarding the ordinary scale, BIE results in a null field in the exterior domain owing to the Green's third identity. It is interesting to find that ordinary and degenerate scales result in a null field in the exterior and interior domains, respectively. To demonstrate this finding, three cases of circle, ellipse and right triangle are demonstrated. Theoretical derivation using the Riemann conformal mapping with the unit logarithmic capacity and the degenerate scale in BEM/BIEM both indicate the null field in the interior domain analytically and numerically.

References

[1] J.T. Chen, J.H. Lin, S.R. Kuo and Y.P. Chiu: Analytical study and numerical experiments for degenerate scale problems in boundary element method using degenerate kernels and circulants, Eng. Anal. Bound. Elem., 25(9), pp. 819-828, 2001.

[2] J.T. Chen, S.R. Kuo and J.H. Lin: Analytical study and numerical experiments for degenerate scale problems in the boundary element method for two-dimensional elasticity, Int. J. Numer. Meth. Eng., 54(12), pp.1669-1681, 2002.

[3] J.T. Chen, C.F. Lee, I.L. Chen and J.H. Lin: An alternative method for degenerate scale problems in boundary element methods for the two-dimensional Laplace equation, Eng. Anal. Bound. Elem., 26(7), pp. 559-569, 2002.

[4] J.T. Chen, W.C. Chen, S.R. Lin and I.L. Chen, Rigid body mode and spurious mode in the dual boundary elemnent formulation for the Laplace equation, Comput. Struct., 81(13), pp. 1395-1404, 2003.

[5] J.T. Chen, S.R. Lin and K.H. Chen: Degenerate scale problem when solving Laplace equation by BEM and its treatment, Int. J. Numer. Meth. Eng., 62(2), pp.233-261, 2005.

[6] J.T. Chen, C.S. Wu, K.H. Chen and Y.T. Lee: Degenerate scale for analysis of circular plate using the boundary integral equations and boundary element method, Comput. Mech., 38, pp 33-49, 2006.

[7] J.T. Chen, Y.T. Lee, S.R. Kuo and Y.W. Chen: Analytical derivation and numerical experiments of degenerate scale for an ellipse in BEM, Eng. Anal. Bound. Elem., 36(9), pp. 1397-1405, 2012.