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STRESS FORMULATION OF COMPLEX VARIABLE BOUNDARY INTEGRAL EQUATION FOR SOLVING TORSION PROBLEMS

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Theory of complex variables is a very powerful mathematical technique for solving two-dimensional problems satisfying the Laplace equation. Based on the Cauchy integral formula, the complex variable boundary integral equation (CVBIE) can be constructed. However, the limitation of the above CVBIE is only suitable for holomorphic (analytic) functions. To solve a harmonic-function pair without satisfying the Cauchy-Riemann equations, we propose a new CVBIE that can be employed to solve any harmonic function in two-dimensional Laplace problems. We can derive the present CVBIE by using the Borel-Pompeiu formula. The difference between the present CVBIE and the conventional CVBIE is that the former one has two boundary integrals instead of only one boundary integral is in the latter one. When the unknown field is a holomorphic (analytic) function, the present CVBIE can be reduced to the conventional CVBIE. To examine the present CVBIE, we consider a torsion problem in this paper since the two shear stress fields satisfy the Laplace equation but do not satisfy the Cauchy-Riemann equations. Based on the present CVBIE, we can straightforward solve the stress fields and the torsional rigidity simultaneously. Finally, several examples, circular bar, elliptical bar, equilateral triangular bar, rectangular bar, asteroid bar and circular bar with keyway, were demonstrated to check the validity of the present method.

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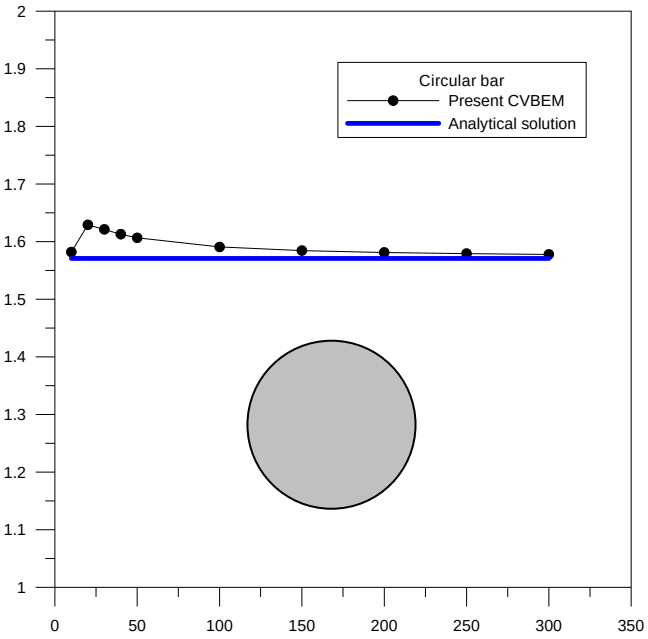

Figure 1. Torsional rigidity versus the number of elements in circular case

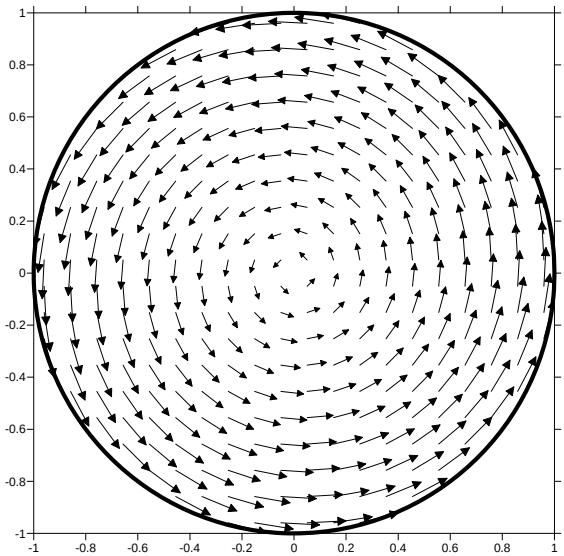

Figure 2. Stress field of the circular case