

On the null and nonzero fields for true and spurious eigenvalues of a prolate spheroidal cavity

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Key words: BIEM, degenerate kernel, Prolate spheroidal coordinates, spheroidal harmonics, null field

Abstract

This talk performs analytical investigation of the true and spurious eigensolutions and eigenmodes of a prolate spheroidal cavity by using the real-part and imaginary-part boundary integral equation method (BIEM). To analytically study the eigenproblems of a prolate spheroidal cavity, the prolate spheroidal coordinates and wave functions are adopted. The fundamental solution is expanded into the degenerate kernel by using the prolate spheroidal coordinates and the boundary densities are expanded by using the spheroidal harmonics. By this way, the boundary contour integral can be analytically determined through the orthogonal relations. Dirichlet and Neumann eigenproblems are both considered. It is interesting to find that the BIEM using the real or the imaginary-part kernel to deal with a prolate spheroidal cavity yields spurious eigensolutions. This finding agrees with those corresponding to the circular and elliptical cases. The true and spurious eigenvalues in the real-part BIEM are found to be the zeros of the radial prolate spheroidal functions of the second kind or their derivatives. Besides, nonzero field in the domain are analytically derived in cases of the true eigenvalues while the interior null field and nonzero field for the complementary domain are also

analytically examined in cases of the spurious eigenvalues.

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