Eigensolution of annular membrane using the method of fundamental solutions

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Abstract

In this paper, the method of fundamental solutions (MFS) for solving the eigenfrequencies of annular membrane is proposed. By employing the fundamental solution, the coefficients of influence matrices are easily determined. The spurious eigensolution in conjunction with the true eigensolution appears. It is found that the spurious eigensolution using the MFS depends on the location of the inner boundary where the sources are distributed. To verify this finding, the true and spurious eigenvalues in an annular domain are analytically studied using the degenerate kernel and circulant. In order to obtain the true eigensolution, the singular value decomposition (SVD) updating technique and the Burton & Miller method are utilized to filter out the spurious eigensolutions. One example is demonstrated analytically and numerically to see the validity of the present method.

Keywords: Annular membrane; Method of fundamental solutions; Helmholtz equation; Circulants; Degenerate kernel; SVD updating technique; Burton and Miller method

1. Introduction

The method of fundamental solutions (MFS) is a numerical technique as well as finite difference method (FDM), finite element method (FEM) and boundary element method (BEM). It is well known that the method of fundamental solutions can deal with many engineering problems when a fundamental solution is known. This method was attributed to Kupradze in 1964 [1]. The method of fundamental solutions can be applied to potential [2], Helmholtz [3], diffusion [4], biharmonic [5] and elasticity problems [1]. The method of fundamental solutions can be seen as one kind of meshless method. The basic idea is to approximate the solution by a linear superposition of fundamental solution with source located outside the domain of the problem. Moreover, it has some advantages over boundary element method, *e.g.*, no singularity, no boundary integrals and mesh-free model.

In boundary element method, Tai and Shaw [6] first employed the complex-valued BEM to solve membrane vibration. De Mey [7], Hutchinson and Wong [8] employed only the real-part kernel to solve the membrane and plate vibrations, respectively. Although the complex-valued computation is avoided, they faced the occurrence of spurious eigenequations. One has to investigate the mode shapes in order to identify and reject the spurious ones. If we usually need to look for the eigenmode as well as eigenvalue, the sorting for the spurious eigenvalues pay a small price by identifying the mode shapes. Chen *et al.* [9] commented that the detection of spurious modes may mislead the judgment of the true and spurious ones, since the true and spurious modes may have the same nodal line in case of different eigenvalues. This is the reason why Chen and his coworkers have developed many systematic techniques, *e.g.*, dual formulation [9], domain partition [10], SVD updating technique [11], CHEEF method [12], for sorting out the true and the spurious eigenvalues. However, it is true only for the case of problem with a simply-connected domain. For multiply-connected problems, spurious eigenvalues still occurs even though the complex-valued BEM is utilized. This occurrence of spurious eigenvalues and their treatment have been studied in the membrane and acoustic problems [13, 14].

Although the MFS has been applied to solve many engineering problems, its validity for solving the eigensolutions of multiply-connected problems was not addressed in the literature to the authors' knowledge. We may wonder whether the spurious solution occur or not as well as BEM does. For the purpose of analytical derivation, an annular case is considered to examine the appearance of spurious eigensolutions.

In this paper, the method of fundamental solutions for solving the eigenfrequencies of annular membrane is proposed. The occurring mechanism of the spurious solution of an annular membrane is studied analytically and numerically. The degenerate kernels and circulants are employed to determine the spurious solution. In order to filter out the spurious eigenvalues, singular value decomposition updating technique and Burton & Miller method are utilized. An annular case is demonstrated analytically and numerically to see the validity of the present method.

2. Formulation of annular problem using method of fundamental solutions

The governing equation for an annular membrane vibration in Fig.1 is the Helmholtz equation as follows:

$$(\nabla^2 + k^2)u(x) = 0, \quad x \in D, \qquad (1)$$

where ∇^2 is the Laplacian operator, D is the domain of interest and k is the wave number.

The fundamental solution $U(s,x)$ which satisfies

$$(\nabla^2 + k^2)U(s,x) = -\frac{\pi}{2}\delta(x-s), \qquad (2)$$

where $\delta(x-s)$ is the Dirac-Delta function. According to the dual formulation [15], we have the four kernels

$$U(s,x) = iJ_0(kr) - Y_0(kr), \qquad (3)$$

$$T(s,x) = \frac{\partial U(s,x)}{\partial n_s} = -k\frac{iJ_1(kr) - Y_1(kr)}{r} y_i n_i, \qquad (4)$$

$$L(s,x) = \frac{\partial U(s,x)}{\partial n_x} = k\frac{iJ_1(kr) - Y_1(kr)}{r} y_i \bar{n}_i, \qquad (5)$$

$$M(s,x) = \frac{\partial^2 U(s,x)}{\partial n_x \partial n_s} = k\Big(\frac{k(-iJ_2(kr) + Y_2(kr))}{r^2} y_i y_j n_i \bar{n}_j + \frac{iJ_1(kr) - Y_1(kr)}{r} n_i \bar{n}_i\Big), \qquad (6)$$

where $r \equiv |s-x|$ is the distance between the source and collocation points; n_i is the ith component of the outnormal vector at s; $\bar{n}_i$ is the ith component of the outnormal vector at x, J_m and Y_m denote the first kind and second kind of the mth order Bessel function, respectively, and $y_i \equiv (s_i - x_i)$, i= 1, 2, are the differences of the ith components of s and x, respectively. Based on the indirect method using the dual formulation, we can represent the field solution by

Single-layer potential approach

$$u(x_i) = \sum_j U(s_j, x_i)\phi_j \ , \tag{7}$$

$$t(x_i) = \sum_j L(s_j, x_i)\phi_j \ , \tag{8}$$

Double-layer potential approach

$$u(x_i) = \sum_j T(s_j, x_i)\psi_j \ , \tag{9}$$

$$t(x_i) = \sum_j M(s_j, x_i)\psi_j \ . \tag{10}$$

The matrix forms of Eqs.(7), (8), (9) and (10) are

Single-layer potential approach

$$\{u_i\} = [U_{ij}]\{\phi_j\}, \tag{11}$$

$$\{t_i\} = [L_{ij}]\{\phi_j\}, \tag{12}$$

Double-layer potential approach

$$\{u_i\} = [T_{ij}]\{\psi_j\}, \tag{13}$$

$$\{t_i\} = [M_{ij}]\{\psi_j\}. \tag{14}$$

where $\{\phi_j\}$ and $\{\psi_j\}$ are the generalized unknowns by using the single and double-layer potential approaches, respectively. Here, we consider the problem with an annular domain. The radii of inner and outer circles are a and b for the real boundary, respectively. The source strengths are distributed on the inner and outer fictitious circular radii a' and b' in Fig.2, respectively. For simplicity, the boundary condition is the Dirichlet type, $\bar{u}=0$ on all the boundaries. We distributed $2N$ collocation points at each real boundary and $2N$ source points at each fictitious boundary. By matching the boundary condition, the equations can be obtained using the single-layer potential approach of Eq.(7) as shown below:

$$\{0\} = \left[U_{ij}^{11}\right]\left\{\phi_j^1\right\} + \left[U_{ij}^{12}\right]\left\{\phi_j^2\right\} \tag{15}$$

$$\{0\} = \left[U_{ij}^{21}\right]\left\{\phi_j^1\right\} + \left[U_{ij}^{22}\right]\left\{\phi_j^2\right\} \tag{16}$$

where the first superscript "α" in $[U_{ij}^{\alpha\beta}]$ denotes the position of collocation point (1 for B_1 and 2 for B_2), the second superscript "β" identifies the position of source point (1 for B_1' and 2 for B_2'), $\left\{\phi_j^1\right\}$ and $\left\{\phi_j^2\right\}$ are the unknow coefficients on the inner and outer boundary, respectively. By assembling Eqs.(15) and (16) together, we have

$$[SM_1]\begin{Bmatrix}\phi_j^1\\ \phi_j^2\end{Bmatrix} = \begin{bmatrix}U_{ij}^{11} & U_{ij}^{12}\\ U_{ij}^{21} & U_{ij}^{22}\end{bmatrix}\begin{Bmatrix}\phi_j^1\\ \phi_j^2\end{Bmatrix} = \begin{Bmatrix}0\\0\end{Bmatrix}. \tag{17}$$

The determinant of the matrix must be zero to obtain the nontrivial solution, *i.e.*,

$$\det[SM_1] = 0 \tag{18}$$

By plotting the determinant versus the wave number, the curve drops at the positions of eigenvalues.

3. Mathematical analysis of the true and spurious eigenvalues

For the kernel function, we can express $x=(\rho,\phi)$ and $s=(R,\theta)$ in terms of polar coordinate. The U kernel can be expressed in terms of degenerate kernels as shown below:

$$U(s,x) = \begin{cases} U^I(\theta,\phi) = \sum_{m=-\infty}^{\infty} J_m(k\rho)[iJ_m(kR) - Y_m(kR)]\cos(m(\theta-\phi)), & R > \rho \\ U^E(\theta,\phi) = \sum_{m=-\infty}^{\infty} J_m(kR)[iJ_m(k\rho) - Y_m(k\rho)]\cos(m(\theta-\phi)), & R < \rho \end{cases} \tag{19}$$

where the subscripts "I" and "E" denote the interior ($R > \rho$) and exterior domains ($R < \rho$), respectively. Since the rotation symmetry is preserved for a circular boundary, the four influence matrices, $[U^{11}]$, $[U^{12}]$, $[U^{21}]$ and $[U^{22}]$ are all symmetric circulants. By superimposing $2N$ lumped strength along each boundary, we have the influence matrices,

$$\left[U^{11}\right] = \begin{bmatrix} a_0 & a_1 & a_2 & \cdots & a_{2N-2} & a_{2N-1} \\ a_{2N-1} & a_0 & a_1 & \cdots & a_{2N-3} & a_{2N-2} \\ a_{2N-2} & a_{2N-1} & a_0 & \cdots & a_{2N-4} & a_{2N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_1 & a_2 & a_3 & \cdots & a_{2N-1} & a_0 \end{bmatrix} \tag{20}$$

where the elements of the first row can be obtained by

$$a_{j-i} = U^{11}(s_j, x_i) \cdot \tag{21}$$

The matrix $[U^{11}]$ in Eq.(20) is found to be a circulant since the rotational symmetry for the influence coefficients is considered. By using the degenerate kernel and the orthogonal property, the eigenvalue of the matrix $[U^{11}]$ can be obtained as follows:

$$\lambda_m^{[U^{11}]} = 2NJ_m(ka')[iJ_m(ka) - Y_m(ka)] \qquad (22)$$

where $m = 0, \pm 1, \pm 2, \cdots, \pm(N-1), N$. Similarly, the eigenvalue of matrices, $[U^{12}]$, $[U^{21}]$ and $[U^{22}]$ are shown in below:

$$\lambda_m^{[U^{12}]} = 2NJ_m(ka)[iJ_m(kb') - Y_m(kb')] \qquad (23)$$

$$\lambda_m^{[U^{21}]} = 2NJ_m(ka')[iJ_m(kb) - Y_m(kb)] \qquad (24)$$

$$\lambda_m^{[U^{22}]} = 2NJ_m(kb)[iJ_m(kb') - Y_m(kb')] \qquad (25)$$

By using the similar transform, we can decompose the $[U^{11}]$ matrix into

$$[U^{11}] = \Phi \Sigma_{[U^{11}]} \Phi^H$$
$$= \Phi \begin{bmatrix} \lambda_0^{[U^{11}]} & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & \lambda_1^{[U^{11}]} & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \lambda_{-1}^{[U^{11}]} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda_{(N-1)}^{[U^{11}]} & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & \lambda_{-(N-1)}^{[U^{11}]} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & \lambda_N^{[U^{11}]} \end{bmatrix} \Phi^T \qquad (26)$$

where "*H*" is transpose and conjugate, and

$$\Phi = \frac{1}{\sqrt{2N}} \begin{bmatrix} 1 & 1 & 0 & \cdots & 1 & 0 & 1 \\ 1 & \cos(\frac{2\pi}{2N}) & \sin(\frac{2\pi}{2N}) & \cdots & \cos(\frac{2\pi(N-1)}{2N}) & \sin(\frac{2\pi(N-1)}{2N}) & \cos(\frac{2\pi N}{2N}) \\ 1 & \cos(\frac{4\pi}{2N}) & \sin(\frac{4\pi}{2N}) & \cdots & \cos(\frac{4\pi(N-1)}{2N}) & \sin(\frac{4\pi(N-1)}{2N}) & \cos(\frac{4\pi N}{2N}) \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 1 & \cos(\frac{2\pi(2N-2)}{2N}) & \sin(\frac{2\pi(2N-2)}{2N}) & \cdots & \cos(\frac{\pi(4N-4)(N-1)}{2N}) & \sin(\frac{\pi(4N-4)(N-1)}{2N}) & \cos(\frac{\pi(4N-4)(N)}{2N}) \\ 1 & \cos(\frac{2\pi(2N-1)}{2N}) & \sin(\frac{2\pi(2N-1)}{2N}) & \cdots & \cos(\frac{\pi(4N-2)(N-1)}{2N}) & \sin(\frac{\pi(4N-2)(N-1)}{2N}) & \cos(\frac{\pi(4N-2)(N)}{2N}) \end{bmatrix} \qquad (27)$$

Similarly, $[U^{12}]$, $[U^{21}]$ and $[U^{22}]$ can be decomposed. Equation (17) can be decomposed into

$$[SM_1] = \begin{bmatrix} \Phi\Sigma_{[U^{11}]}\Phi^H & \Phi\Sigma_{[U^{12}]}\Phi^H \\ \Phi\Sigma_{[U^{21}]}\Phi^H & \Phi\Sigma_{[U^{22}]}\Phi^H \end{bmatrix},$$
$$= \begin{bmatrix} \Phi & 0 \\ 0 & \Phi \end{bmatrix} \begin{bmatrix} \Sigma_{[U^{11}]} & \Sigma_{[U^{12}]} \\ \Sigma_{[U^{21}]} & \Sigma_{[U^{22}]} \end{bmatrix} \begin{bmatrix} \Phi & 0 \\ 0 & \Phi \end{bmatrix}^H \qquad (28)$$

Since Φ is unitary, the determinant of $[SM_1]$ is

$$\det[SM_1] = \sigma_0(\sigma_1\sigma_2\cdots\sigma_{N-1})^2\sigma_N = 0, \qquad (29)$$

where

$$\sigma_m = \lambda_m^{[U^{11}]}\lambda_m^{[U^{22}]} - \lambda_m^{[U^{12}]}\lambda_m^{[U^{21}]}$$
$$= 4N^2 J_m(ka')[-iJ_m(kb') + Y_m(kb')] \qquad (30)$$
$$\{J_m(kb)Y_m(ka) - J_m(ka)Y_m(kb)\}$$

for the annular membrane with the Dirichlet-Dirichlet boundary conditions by using the single-layer potential approach. After comparing with the analytical solution [14], we can obtain the true and spurious eigenequations in Eq.(29). Since, the middle bracket $[-iJ_m(kb') + Y_m(kb')]$ of Eq.(30) is never zero, the spurious eigenequation of $J_m(ka') = 0$ and the true eigenequation $J_m(kb)Y_m(ka) - J_m(ka)Y_m(kb) = 0$ and the spurious eigenequation of $J_m(ka') = 0$. Similarly, we can obtain the true and spurious eigenequation for different boundary conditions and using different formulations. All the results are derived analytically as shown in Table 1. The occurrence of spurious eigenvalues depends on the formulation and the location of inner source point instead of the specified boundary condition, while the true eigenequation is independent of the formulation and is relevant to the specified boundary condition. For the multiply-connected membrane, the singular-layer potential approach produces spurious eigenvalues which are associated with the interior natural frequency with essential homogeneous boundary conditions, while the double-layer potential approach produces spurious eigenvalues which are associated with the interior eigenfrequency with natural homogeneous boundary conditions.

4. Treatment of spurious eigenvalues

4.1 SVD updating technique

In order to extract out the true eigenvalues, the SVD updating technique is utilized. In spite of the single-layer potential approach to obtain Eq.(17), we can also select the double-layer potential approach and obtain

$$[SM_2]\begin{Bmatrix} \psi_j^1 \\ \psi_j^2 \end{Bmatrix} = \begin{bmatrix} T^{11} & T^{12} \\ T^{21} & T^{22} \end{bmatrix} \begin{Bmatrix} \psi_j^1 \\ \psi_j^2 \end{Bmatrix} = \{0\}. \qquad (31)$$

By employing the relation in the degenerate kernels between direct and indirect method [16], the SVD updating document (Indirect method) to extract out the true eigenequation is equivalent to the SVD updating document (Indirect method). We have

$$[C] = \begin{bmatrix} (SM_1)^H \\ (SM_2)^H \end{bmatrix}, \tag{32}$$

Since the rank of the matrix $[C]$ must be smaller than *4N*. By using the property of Eq.(26), the matrix can be written as

$$[C] = \begin{bmatrix} \Phi & 0 & 0 & 0 \\ 0 & \Phi & 0 & 0 \\ 0 & 0 & \Phi & 0 \\ 0 & 0 & 0 & \Phi \end{bmatrix} \begin{bmatrix} \Sigma_{U^{11}} & \Sigma_{U^{21}} \\ \Sigma_{U^{12}} & \Sigma_{U^{22}} \\ \Sigma_{T^{11}} & \Sigma_{T^{21}} \\ \Sigma_{T^{12}} & \Sigma_{T^{22}} \end{bmatrix} \begin{bmatrix} \Phi^{-1} & 0 \\ 0 & \Phi^{-1} \end{bmatrix} \tag{33}$$

Based on the equivalence between the SVD technique and the least-squares method, we can obtain the true eigenequation ($J_m(kb)Y_m(ka) - \ J_m(ka)Y_m(kb) = 0$). This indicates that only the true eigenvalues for annular membrane are imbedded in the SVD updating matrix.

4.2 Burton & Miller method

By employing the Burton a& Miller method for dealing with fictitious frequency, we extend this concept to suppress the appearance of the spurious eigenvalue of the annular membrane in the method of fundamental solutions.

By assembling the Eqs.(17) and (31) with an imaginary number, we have

$$\left[[SM_1] + i[SM_2]\right]\begin{Bmatrix} \varphi_1 \\ \varphi_2 \end{Bmatrix} = \{0\}, \tag{34}$$

where the φ_1 and φ_2 are the mixed densities. Thus, the true eigenequation ($J_m(kb)Y_m(ka) - J_m(ka)Y_m(kb) = 0$) is obtained by using the Burton & Miller method.

5. Numerical example

An annular membrane with the inner radius of 0.5 meter and the outer radius of 2 meter are considered, respectively. The source points are distributed at $a' = 0.4\ m$ and $b' = 2.2\ m$. The outer and inner fictitious boundaries are both distributed 36 nodes as shown in Fig.2, respectively. Fig.3 and Fig.4 show the determinant versus wave number by using the single-layer potential approach and double-layer potential approach, respectively. The drop location indicates the possible eigenvalues. As expected, the spurious eigenvalue of *k*=6.01 ($J_m(ka') = 0$) for single-layer potential approach and *k*=4.61 ($J'_m(ka') = 0$) for double-layer potential approach appear. Fig.5 shows the determinant versus wave number by using the SVD updating technique. Fig.6 shows the determinant versus wave number by using the Burton & Miller method for annular membrane. It is found that the spurious eigenvalues are suppressed. The former five true eigenvalues using the method of fundamental solutions are compared with those using FEM and BEM as shown in Table 2. After comparing the result with the analytical solution, good agreement is made.

6. Conclusions

The mathematical analysis has shown that the spurious eigenvalues occur by using degenerate kernel and circulants when the method of fundamental solutions is used. The positions of spurious eigenvalues for the annular problem depend on the location of inner fictitious boundary where the sources are distributed. The spurious eigenvalues in the annular problem are found to be the true eigenvalues of the associated simply-connected problem boundary by inner boundary. We have employed the SVD updating technique and Burton & Miller method to filter out the spurious eigenvalues successfully.

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Table 1 The true and spurious eigenequation for different boundary conditions by using single-layer and double-layer potential approaches.

Inner-outer boundary		Single-layer potential approach	Double-layer potential approach
Dirichlet-Dirichlet	*True*	$J_m(kb)Y_m(ka) - J_m(ka)Y_m(kb) = 0$	$J_m(kb)Y_m(ka) - J_m(ka)Y_m(kb) = 0$
	Spurious	$J_m(ka') = 0$	$J'_m(ka') = 0$
Dirichlet-Neumann	*True*	$J'_m(kb)Y_m(ka) - J_m(ka)Y'_m(kb) = 0$	$J'_m(kb)Y_m(ka) - J_m(ka)Y'_m(kb) = 0$
	Spurious	$J_m(ka') = 0$	$J'_m(ka') = 0$
Neumann-Dirichlet	*True*	$J_m(kb)Y'_m(ka) - J'_m(ka)Y_m(kb) = 0$	$J_m(kb)Y'_m(ka) - J'_m(ka)Y_m(kb) = 0$
	Spurious	$J_m(ka') = 0$	$J'_m(ka') = 0$
Neumann-Neumann	*True*	$J'_m(kb)Y'_m(ka) - J'_m(ka)Y'_m(kb) = 0$	$J'_m(kb)Y'_m(ka) - J'_m(ka)Y'_m(kb) = 0$
	Spurious	$J_m(ka') = 0$	$J'_m(ka') = 0$

Table 2 The former five true eigenvalues are compared with different methods

	k_1	k_2	k_3	k_4	k_5
Analysis solution [14]	**2.05**	**2.23**	**2.66**	**3.21**	**3.80**
FEM (ABAQUS) [14]	**2.03**	**2.20**	**2.62**	**3.15**	**3.71**
BEM (CHIEF) [14]	**2.05**	**2.23**	**2.67**	**3.22**	**3.81**
MFS (SVD updating technique)	**2.05**	**2.22**	**2.65**	**3.20**	**3.79**
MFS (Burton & Miller)	**2.04**	**2.20**	**2.64**	**3.20**	**3.78**

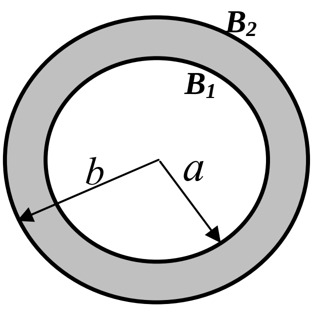

Fig. 1 An annular problem

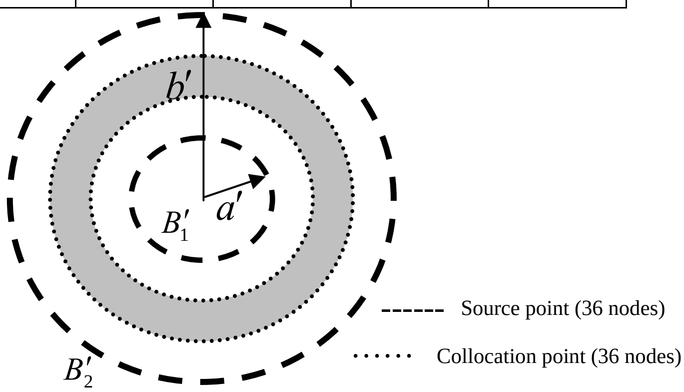

Fig. 2 Figure sketch for node distribution

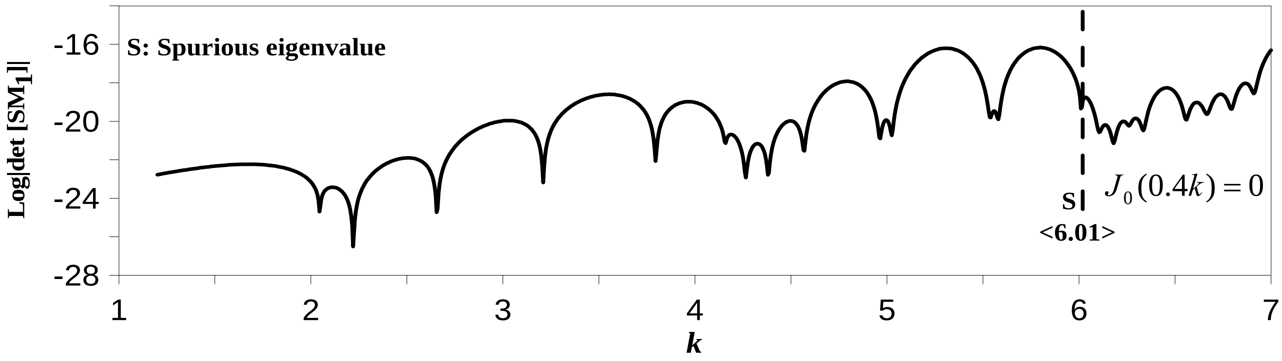

Fig.3 The determinant versus the wave number by using the single-layer potential approach

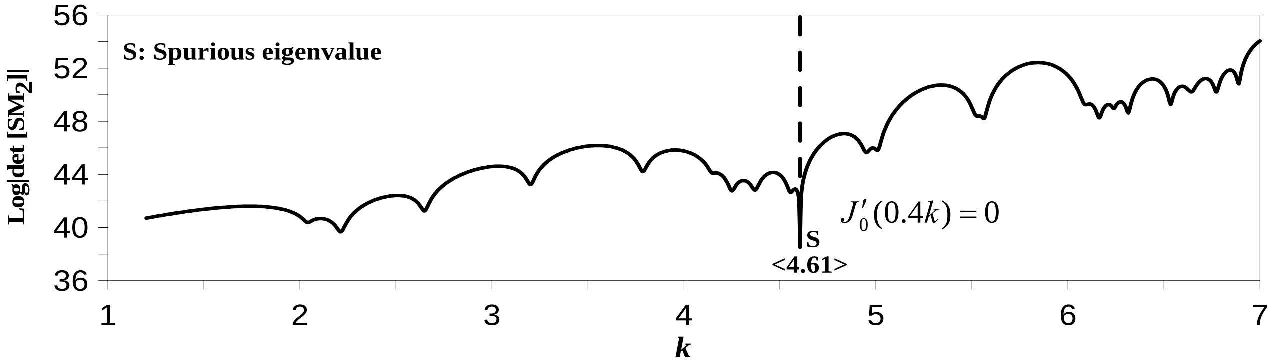

Fig.4 The determinant versus the wave number by the using double-layer potential approach

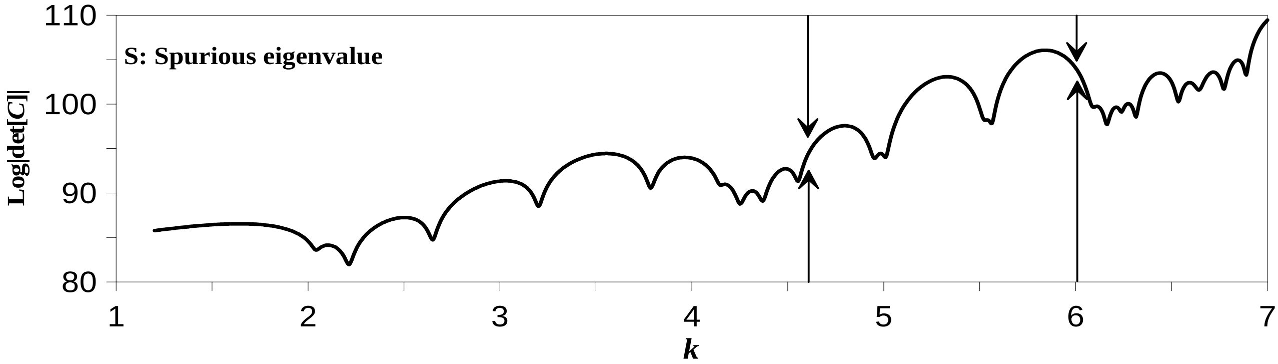

Fig.5 The determinant versus the wave number by using the SVD updating technique

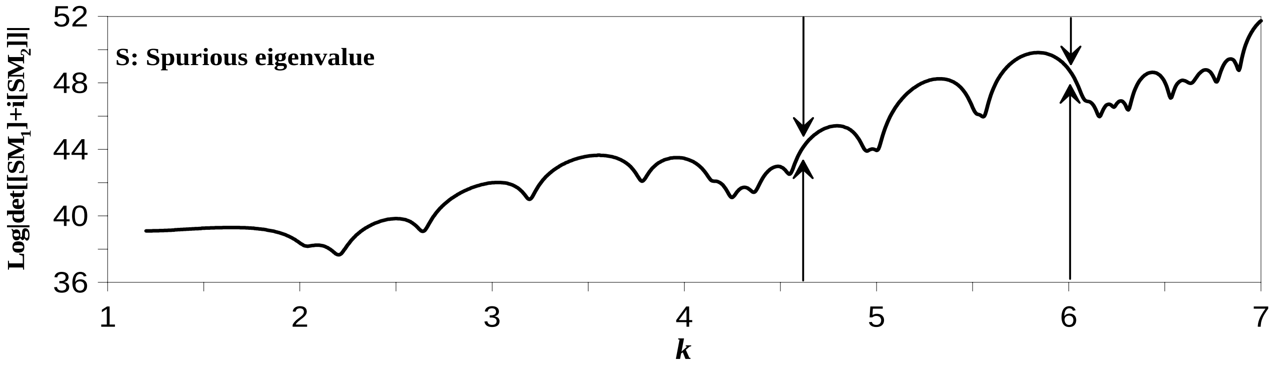

Fig.6 The determinant versus the wave number by using the Burton and Miller method

基本解法求同心圓薄膜之特徵解

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摘要

本文主要是用基本解法求解同心圓薄膜之特徵值問題。利用基本解法，我們可以容易地求得影響係數矩陣。在求解的過程中，假根常常伴隨著真根出現，我們也發現到假根產生的機制是受到內部源點分佈位置的影響。為了證明這個發現，我們將以一個同心圓薄膜為例進行解析探討及數值結果的比較，其中我們將會使用退化核及循環矩陣這兩個數學的工具來進行分析。為了獲得真根，採用奇異值補充行及 Burton & Miller 法這兩個技巧來濾除假根。文中以一個例子來論證我們這個方法的正確性。

關鍵字：同心圓薄膜、基本解法、Helmholtz 方程、循環矩陣、退化核、奇異值補充行、Burton & Miller 法