A semi-analytical approach for solving surface motion of multiple alluvial valleys for incident plane SH-waves

半解析法求解多山谷沉積物入射SH波之地表位移

Reporter: 陳柏源

Authors : 陳柏源、陳正宗博士

國立台灣海洋大學河海工程學系

Q dwlrqdo#Wdlzdq#Rfhdq#Xqlyhuvlw|

G hsduwp hqw#ri#K dueru#dqg#U lyhu

Outlines

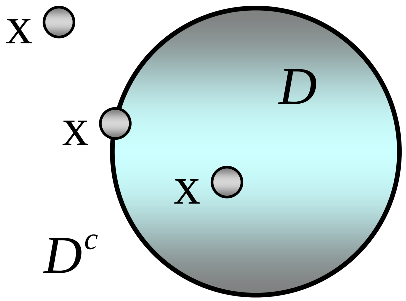
- Motivation
- Present method
  - Expansions of fundamental solution and boundary density
  - Adaptive observer system
  - Linear algebraic equation
  - Image technique for solving scattering problems of half-plane
- Numerical examples
- Conclusions

Outlines

- Motivation
- Present method
  - Expansions of fundamental solution and boundary density
  - Adaptive observer system
  - Linear algebraic equation
  - Image technique for solving scattering problems of half-plane
- Numerical examples
- Conclusions

Conventional BEM

Interior case

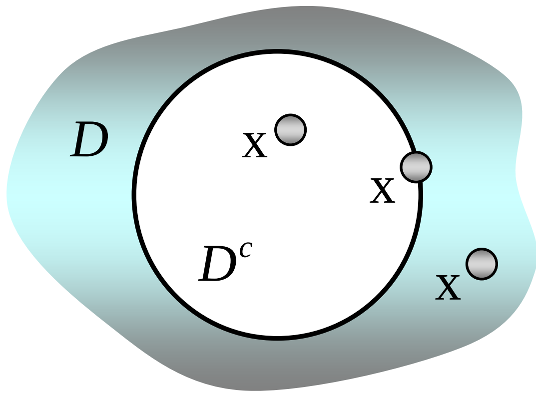


$$U(\mathrm{s},\mathrm{x}) = \ln|\mathrm{x}-\mathrm{s}| = \ln r$$

$$T(\mathrm{s},\mathrm{x}) = \frac{\partial U(\mathrm{s},\mathrm{x})}{\partial \mathrm{n}_\mathrm{s}}$$

$$\psi(\mathrm{s}) = \frac{\partial \varphi(\mathrm{s})}{\partial \mathrm{n}_\mathrm{s}}$$

Exterior case

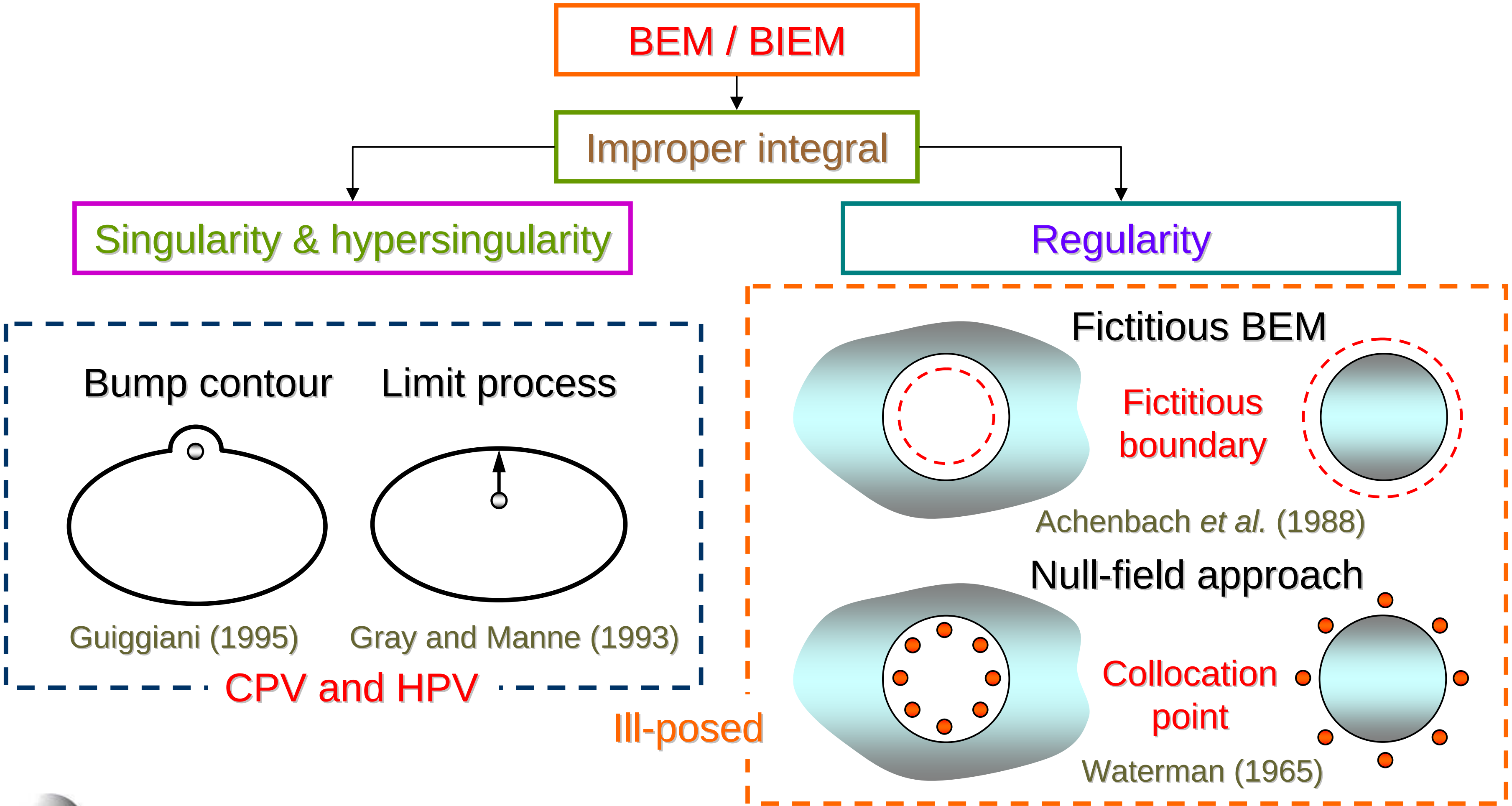


$$2\pi\varphi(\mathrm{x}) = \int_B T(\mathrm{s},\mathrm{x})\varphi(\mathrm{s})dB(\mathrm{s}) - \int_B U(\mathrm{s},\mathrm{x})\psi(\mathrm{s})dB(\mathrm{s}), \quad \mathrm{x} \in D \cup B$$

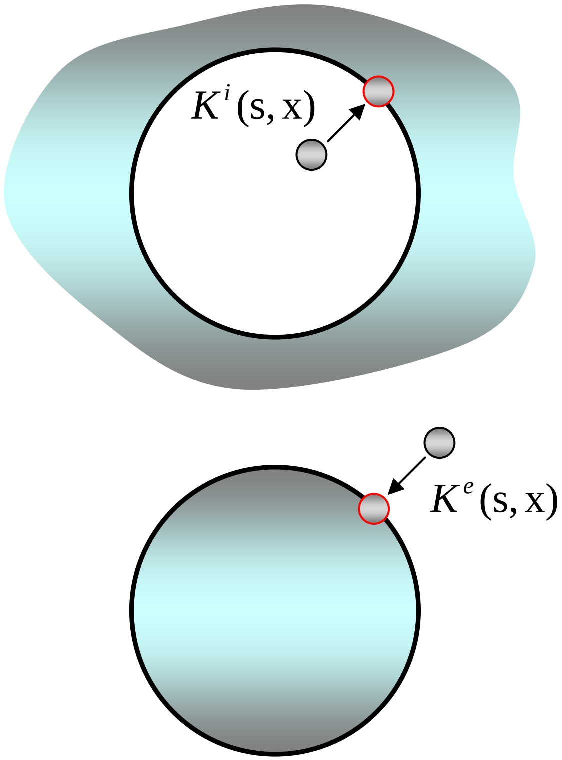
$$\pi\varphi(\mathrm{x}) = C.P.V.\int_B T(\mathrm{s},\mathrm{x})\varphi(\mathrm{s})dB(\mathrm{s}) - R.P.V.\int_B U(\mathrm{s},\mathrm{x})\psi(\mathrm{s})dB(\mathrm{s}), \quad \mathrm{x} \in B$$

$$0 = \int_B T(\mathrm{s},\mathrm{x})\varphi(\mathrm{s})dB(\mathrm{s}) - \int_B U(\mathrm{s},\mathrm{x})\psi(\mathrm{s})dB(\mathrm{s}), \quad \mathrm{x} \in D^c \cup B$$

Motivation


BEM / BIEM
Improper integral
Singularity & hypersingularity
Regularity
Bump contour
Limit process
Guiggiani (1995)
Gray and Manne (1993)
CPV and HPV
Ill-posed
Fictitious BEM
Fictitious boundary
Achenbach et al. (1988)
Null-field approach
Collocation point
Waterman (1965)

Present method



$$\varphi(x) = \int_B K(s,x)\psi(s)dB(s)$$

Degenerate kernel

$$\begin{cases} K^i(s,x), & |s| \ge |x| \\ K^i(s,x), & |x| > |s| \end{cases}$$

→ No principal value

Fundamental solution

$$\ln|x-s|$$

→ CPV and HPV

Advantages of degenerate kernel

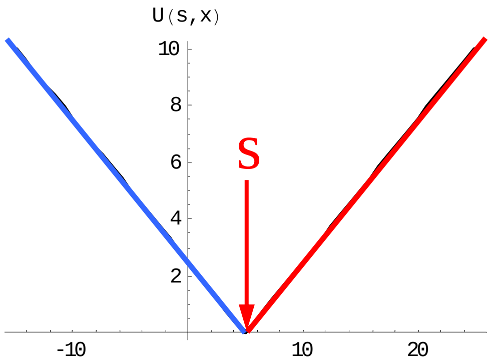
1. No principal value
2. Well-posed
3. Exponential convergence
4. Free of boundary-layer effect

Outlines

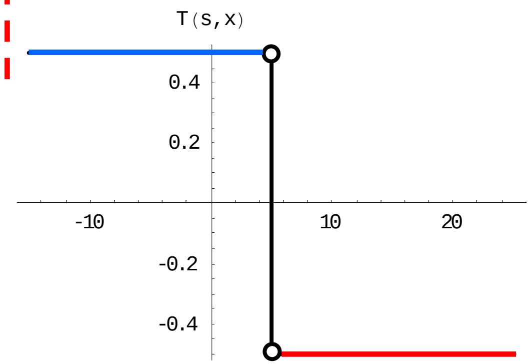
- Motivation
- Present method
  - Expansions of fundamental solution and boundary density
  - Adaptive observer system
  - Linear algebraic equation
  - Image technique for solving scattering problems of half-plane
- Numerical examples
- Conclusions

Separable form of fundamental solution (1D)

Separable property $U(\mathrm{s},\mathrm{x}) = \begin{cases} \sum_{i=1}^{2} a_i(\mathrm{x}) b_i(\mathrm{s}), & \mathrm{s} \geq \mathrm{x} \\ \sum_{i=1}^{2} a_i(\mathrm{s}) b_i(\mathrm{x}), & \mathrm{x} > \mathrm{s} \end{cases}$



continuous



jump

$$U(\mathrm{s},\mathrm{x}) = \frac{1}{2} r = \begin{cases} \frac{1}{2}(\mathrm{s}-\mathrm{x}), & \mathrm{s} \geq \mathrm{x} \\ \frac{1}{2}(\mathrm{x}-\mathrm{s}), & \mathrm{x} > \mathrm{s} \end{cases}$$

$$T(\mathrm{s},\mathrm{x}) = \begin{cases} \frac{1}{2}, & \mathrm{s} > \mathrm{x} \\ \frac{-1}{2}, & \mathrm{x} > \mathrm{s} \end{cases}$$

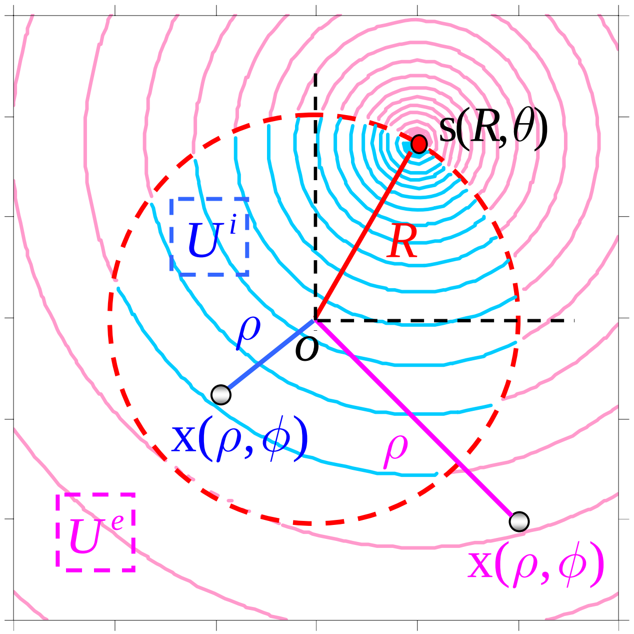
Degenerate (separate) form of fundamental solution (2-D)

$$U(\mathrm{s},\mathrm{x}) = -i\pi H_0^{(1)}(kr)/2 = \begin{cases} U^i(R,\theta;\rho,\phi) = \dfrac{-\pi i}{2}\displaystyle\sum_{m=0}^{\infty} J_m(k\rho)H_m^{(1)}(kR)\cos(m(\theta-\phi)), & R \geq \rho \\ U^e(R,\theta;\rho,\phi) = \dfrac{-\pi i}{2}\displaystyle\sum_{m=0}^{\infty} H_m^{(1)}(k\rho)J_m(kR)\cos(m(\theta-\phi)), & \rho > R \end{cases}$$

$$T(\mathrm{s},\mathrm{x}) \equiv \frac{\partial U(\mathrm{s},\mathrm{x})}{\partial \mathrm{n_s}}$$

$$L(\mathrm{s},\mathrm{x}) \equiv \frac{\partial U(\mathrm{s},\mathrm{x})}{\partial \mathrm{n_x}}$$

$$M(\mathrm{s},\mathrm{x}) \equiv \frac{\partial^2 U(\mathrm{s},\mathrm{x})}{\partial \mathrm{n_s}\,\partial \mathrm{n_x}}$$



$$\mathrm{U(s, x)} \xrightarrow{\quad \frac{\partial}{\partial \mathrm{n_s}} \quad} \mathrm{T(s, x)}$$

$$U(\mathrm{s},\mathrm{x}) = \begin{cases} U^{i}(\mathrm{s},\mathrm{x}) = \sum_{i=1}^{\infty} a_i(\mathrm{x}) b_i(\mathrm{s}), & \mathrm{s} \ge \mathrm{x} \\ U^{e}(\mathrm{s},\mathrm{x}) = \sum_{i=1}^{\infty} a_i(\mathrm{s}) b_i(\mathrm{x}), & \mathrm{x} > \mathrm{s} \end{cases}$$

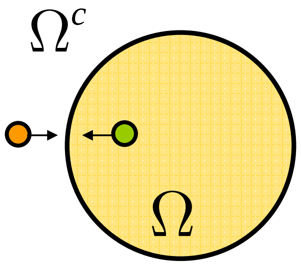
$$T(\mathrm{s},\mathrm{x}) = \begin{cases} T^{i}(\mathrm{s},\mathrm{x}) = \sum_{i=1}^{\infty} a_i(\mathrm{x}) b_i'(\mathrm{s}), & \mathrm{s} \ge \mathrm{x} \\ T^{e}(\mathrm{s},\mathrm{x}) = \sum_{i=1}^{\infty} a_i'(\mathrm{s}) b_i(\mathrm{x}), & \mathrm{x} > \mathrm{s} \end{cases}$$

$$T^{i}(\mathrm{s},\mathrm{x}) - T^{e}(\mathrm{s},\mathrm{x}) = \sum_{i=1}^{\infty} a_i(\mathrm{x}) b_i'(\mathrm{s}) - a_i'(\mathrm{s}) b_i(\mathrm{x})$$

$$\xrightarrow{\mathrm{x} \Rightarrow \mathrm{s}} W\left(a_i(\mathrm{s}), b_i(\mathrm{s})\right)$$

$$T(s,x) = \begin{cases} T^{i}(s,x) = \dfrac{-\pi k i}{2} \sum_{m=0}^{\infty} \varepsilon_m J_m(k\rho) \left[Y_m'(kR) - i J_m'(kR) \right] \cos(m(\theta - \phi)), R > \rho \\ T^{e}(s,x) = \dfrac{-\pi k i}{2} \sum_{m=0}^{\infty} \varepsilon_m J_m'(kR) \left[Y_m(k\rho) - i J_m(k\rho) \right] \cos(m(\theta - \phi)), \rho > R \end{cases}$$

Jump behavior across the boundary



$$2\pi u(\mathrm{x}) = \int_B T^i(\mathrm{s},\mathrm{x})u(\mathrm{s})dB(\mathrm{s}) - \int_B U^i(\mathrm{s},\mathrm{x})t(\mathrm{s})dB(\mathrm{s}) \qquad \mathrm{x} \in \Omega \cup B$$

$$0 = \int_B T^e(\mathrm{s},\mathrm{x})u(\mathrm{s})dB(\mathrm{s}) - \int_B U^e(\mathrm{s},\mathrm{x})t(\mathrm{s})dB(\mathrm{s}) \qquad \mathrm{x} \in \Omega^c \cup B$$

$$\int_0^{2\pi} \left(U^i(\mathrm{s},\mathrm{x}) - U^e(\mathrm{s},\mathrm{x})\right)\cos(n\theta)Rd\theta$$
$$= R\pi^2 J_n(kR)\left[Y_n(kR) - iJ_n(kR)\right]\cos(n\phi) - R\pi^2 J_n(kR)\left[Y_n(kR) - iJ_n(kR)\right]\cos(n\phi)$$
$$= 0, \qquad \mathrm{x} \in B$$

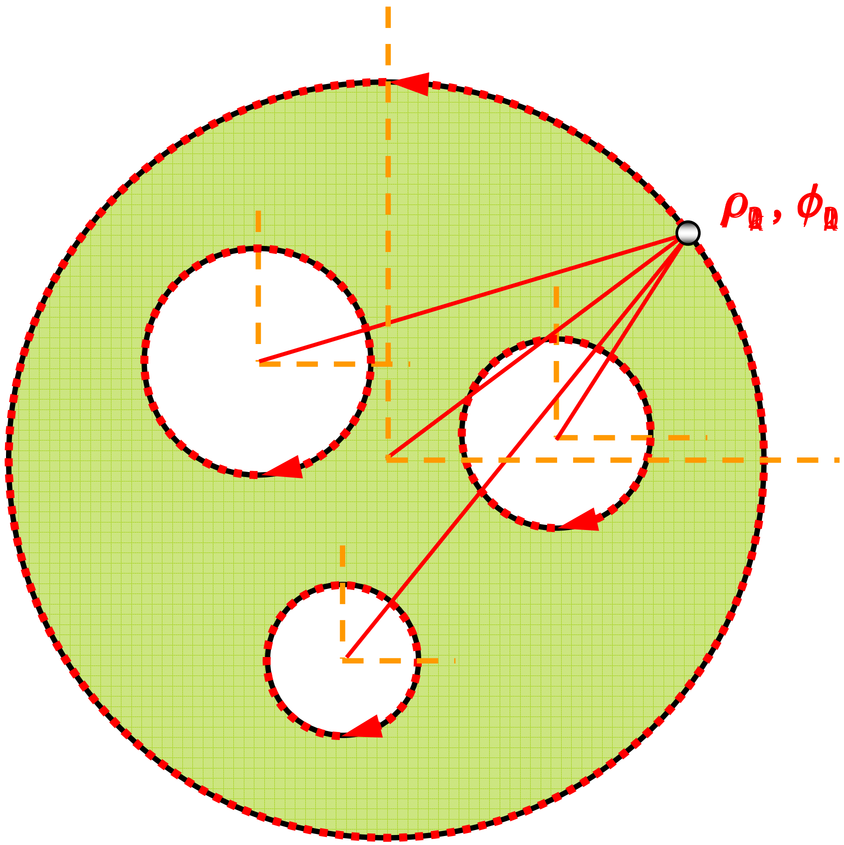
$$\int_0^{2\pi} \left(T^i(\mathrm{s},\mathrm{x}) - T^e(\mathrm{s},\mathrm{x})\right)\cos(n\theta)Rd\theta$$
$$= kR\pi^2 J_n(kR)\left[Y_n'(kR) - iJ_n'(kR)\right]\cos(n\phi) - kR\pi^2 J_n'(kR)\left[Y_n(kR) - iJ_n(kR)\right]\cos(n\phi)$$
$$= kR\left[Y_m'(kR)J_m(kR) - Y_m(kR)J_m'(kR)\right]\cos(n\phi)$$
$$= 2\pi\cos(n\phi), \qquad \mathrm{x} \in B$$

$$W\left(J_m(kR), Y_m(kR)\right) = Y_m'(kR)J_m(kR) - Y_m(kR)J_m'(kR) = \frac{2}{\pi kR}$$

Outlines

- Motivation
- Present method
  - Expansions of fundamental solution and boundary density
  - Adaptive observer system
  - Linear algebraic equation
  - Image technique for solving scattering problems of half-plane
- Numerical examples
- Conclusions

Adaptive observer system



○ collocation point

Outlines

- Motivation
- Present method
  - Expansions of fundamental solution and boundary density
  - Adaptive observer system
  - Linear algebraic equation
  - Image technique for solving scattering problems of half-plane
- Numerical examples
- Conclusions

Linear algebraic equation

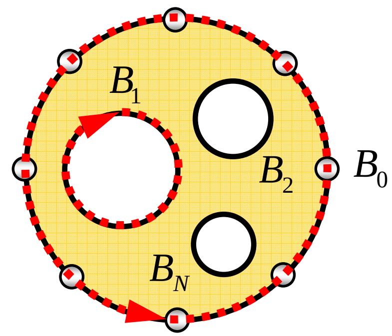
$$[\mathbf{U}]\{\psi\} = [\mathbf{T}]\{\varphi\}$$

Index of collocation circle

$$[\mathbf{U}] = \begin{bmatrix} \mathbf{U}_{00} & \mathbf{U}_{01} & \cdots & \mathbf{U}_{0N} \\ \mathbf{U}_{10} & \mathbf{U}_{11} & \cdots & \mathbf{U}_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{U}_{N0} & \mathbf{U}_{N1} & \cdots & \mathbf{U}_{NN} \end{bmatrix} \qquad \{\psi\} = \begin{Bmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{Bmatrix}$$

Index of routing circle

Column vector of Fourier coefficients
(Nth routing circle)



Explicit form of each submatrix and vector

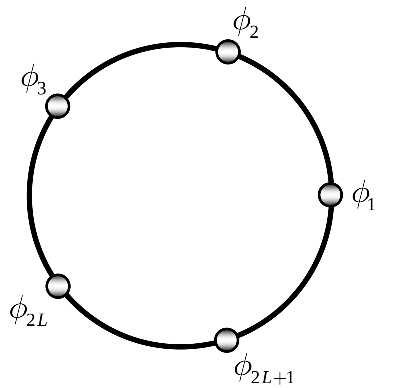
Truncated terms of Fourier series

$$
[\mathbf{U}_{jk}] = \begin{bmatrix}
U_{jk}^{0c}(\phi_1) & U_{jk}^{1c}(\phi_1) & U_{jk}^{1s}(\phi_1) & \cdots & U_{jk}^{Lc}(\phi_1) & U_{jk}^{Ls}(\phi_1) \\
U_{jk}^{0c}(\phi_2) & U_{jk}^{1c}(\phi_2) & U_{jk}^{1s}(\phi_2) & \cdots & U_{jk}^{Lc}(\phi_2) & U_{jk}^{Ls}(\phi_2) \\
U_{jk}^{0c}(\phi_3) & U_{jk}^{1c}(\phi_3) & U_{jk}^{1s}(\phi_3) & \cdots & U_{jk}^{Lc}(\phi_3) & U_{jk}^{Ls}(\phi_3) \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
U_{jk}^{0c}(\phi_{2L}) & U_{jk}^{1c}(\phi_{2L}) & U_{jk}^{1s}(\phi_{2L}) & \cdots & U_{jk}^{Lc}(\phi_{2L}) & U_{jk}^{Ls}(\phi_{2L}) \\
U_{jk}^{0c}(\phi_{2L+1}) & U_{jk}^{1c}(\phi_{2L+1}) & U_{jk}^{1s}(\phi_{2L+1}) & \cdots & U_{jk}^{Lc}(\phi_{2L+1}) & U_{jk}^{Ls}(\phi_{2L+1})
\end{bmatrix}
$$

Number of collocation points

$$
\{\psi_k\} = \{ p_0^k \quad p_1^k \quad q_1^k \quad \cdots \quad p_L^k \quad q_L^k \}^T
$$

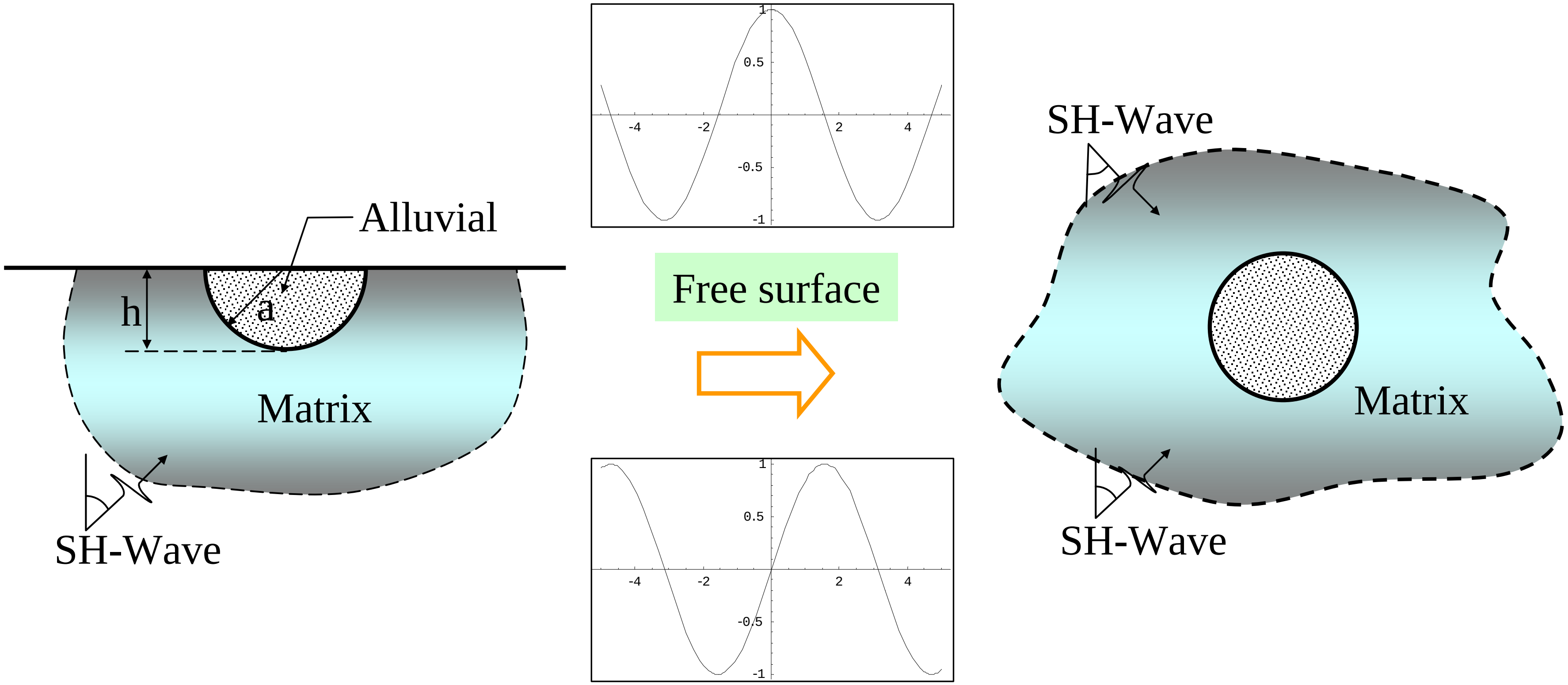
Fourier coefficients



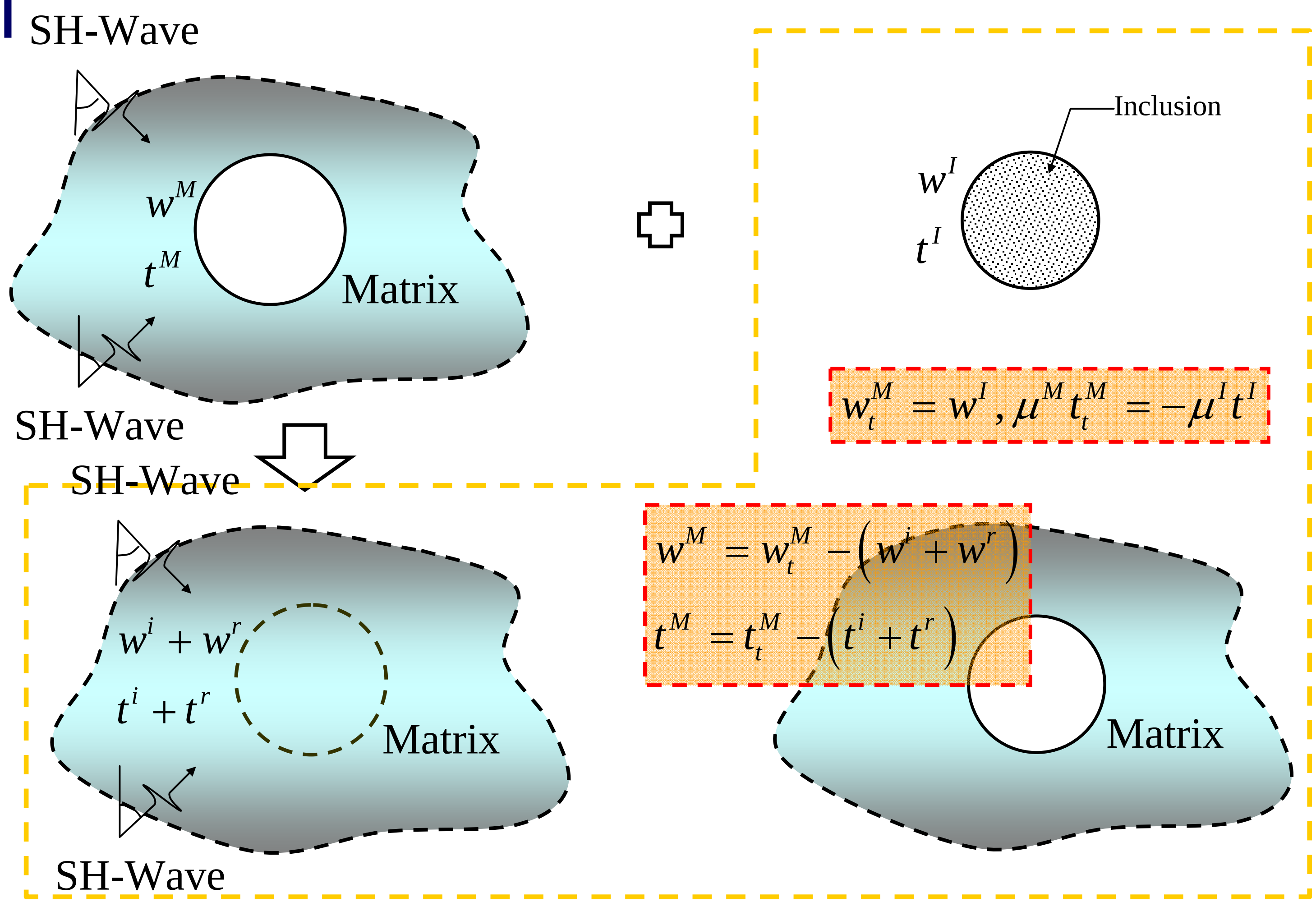
Outlines

- Motivation
- Present method
  - Expansions of fundamental solution and boundary density
  - Adaptive observer system
  - Linear algebraic equation
  - Image technique for solving scattering problems of half-plane
- Numerical examples
- Conclusions

Image technique for solving half-plane problem



Take free body



Linear algebraic system of the inclusion problem

- Matrix field

$$\left[\mathbf{U}^M\right]\left\{\mathbf{t}_t^M-\mathbf{t}^{i+r}\right\}=\left[\mathbf{T}^M\right]\left\{\mathbf{u}_t^M-\mathbf{u}^{i+r}\right\}$$

- Inclusion field

$$\left[\mathbf{U}^I\right]\left\{\mathbf{t}^I\right\}=\left[\mathbf{T}^I\right]\left\{\mathbf{u}^I\right\}$$

- Two constrains

$$\left\{\mathbf{u}_t^M\right\}=\left\{\mathbf{u}^I\right\}$$

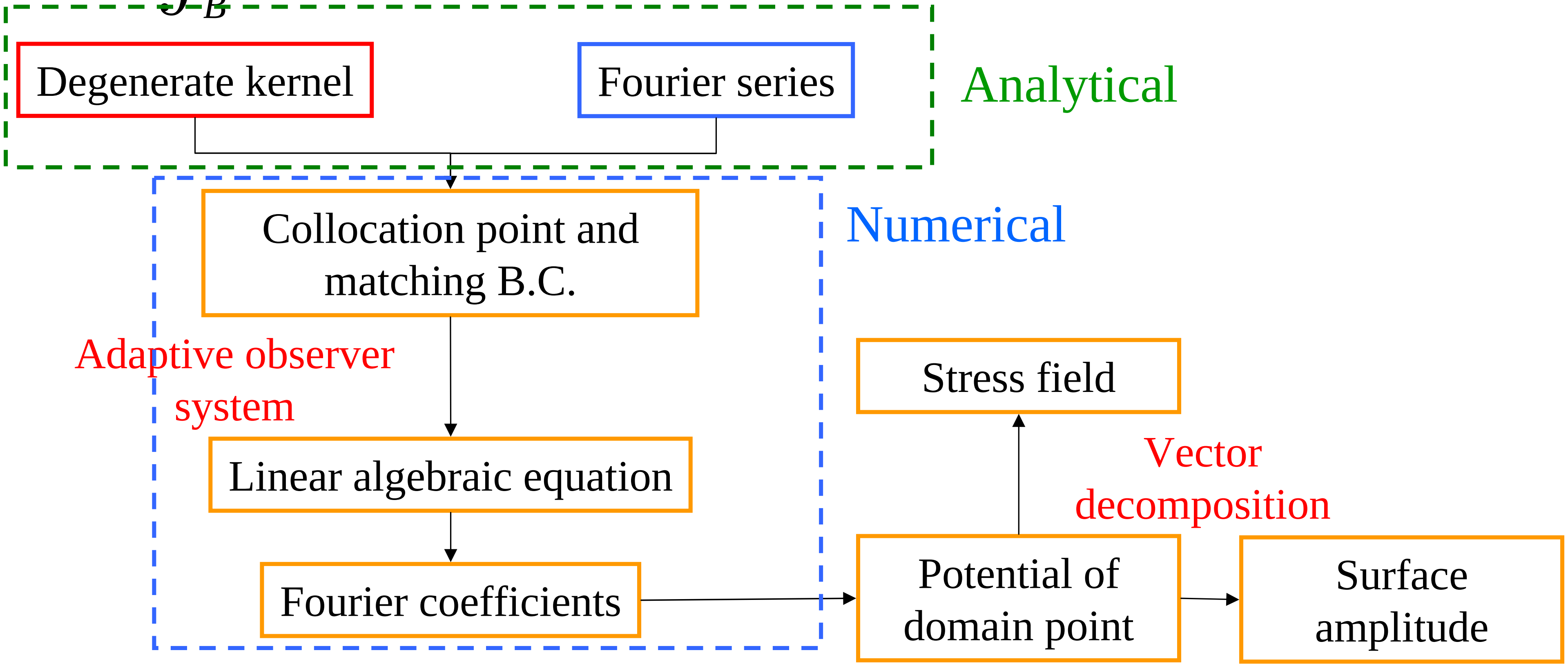
$$\left[\boldsymbol{\mu}^M\right]\left\{\mathbf{t}_t^M\right\}=-\left[\boldsymbol{\mu}^I\right]\left\{\mathbf{t}^I\right\}$$

$$\Rightarrow\begin{bmatrix}\mathbf{T}^M & -\mathbf{U}^M & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{T}^I & -\mathbf{U}^I\\ \mathbf{I} & \mathbf{0} & -\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\mu}^M & \mathbf{0} & \boldsymbol{\mu}^I\end{bmatrix}\begin{Bmatrix}\mathbf{u}_t^M\\ \mathbf{t}_t^M\\ \mathbf{u}^I\\ \mathbf{t}^I\end{Bmatrix}=\begin{Bmatrix}\mathbf{u}(\mathbf{x})^{i+r}\\ \mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\end{Bmatrix}$$

$$\left\{\mathbf{u}(\mathbf{x})^{i+r}\right\}=\left\langle\mathbf{T}^M \quad -\mathbf{U}^M\right\rangle\begin{Bmatrix}\mathbf{u}^{i+r}\\ \mathbf{t}^{i+r}\end{Bmatrix}$$

Flowchart of present method

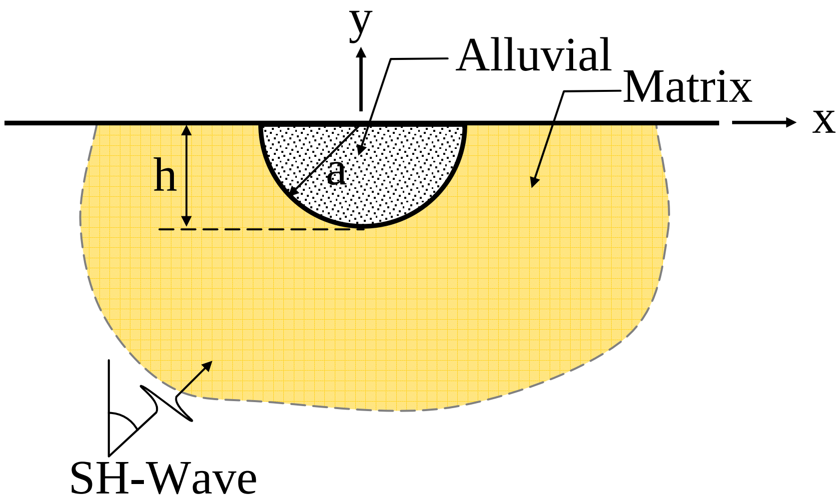
$$0=\int_B [T(s,x)u(s)-U(s,x)t(s)]dB(s)$$



Outlines

- Motivation
- Present method
  - Expansions of fundamental solution and boundary density
  - Adaptive observer system
  - Linear algebraic equation
  - Image technique for solving scattering problems of half-plane
- Numerical examples
- Conclusions

A half-plane problem with a semi-circular alluvial valley subject to the SH-wave



Governing equation

$$(\nabla^2 + k^2)w(x) = 0, \quad x \in \Omega$$

Wave number

$$k = \frac{\pi\eta}{ac}$$

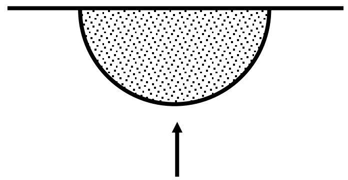
Dimensionless frequency

$$\eta = \frac{\omega a}{\pi}$$

Velocity of shear wave

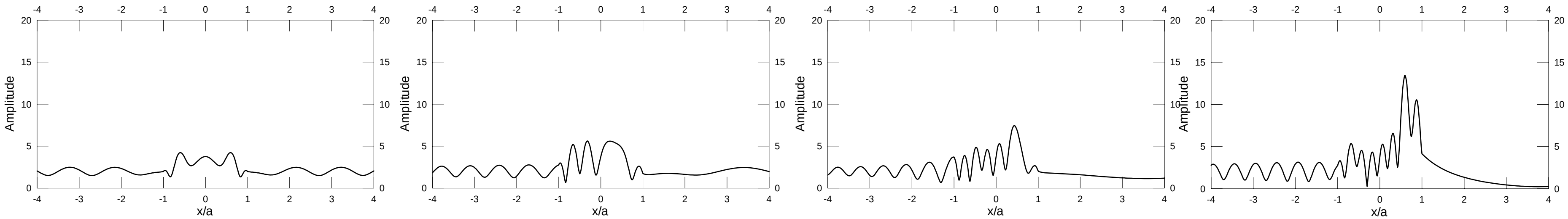
$$c = \sqrt{\frac{\mu}{\rho}}$$

Surface amplitudes of the alluvial valley problem

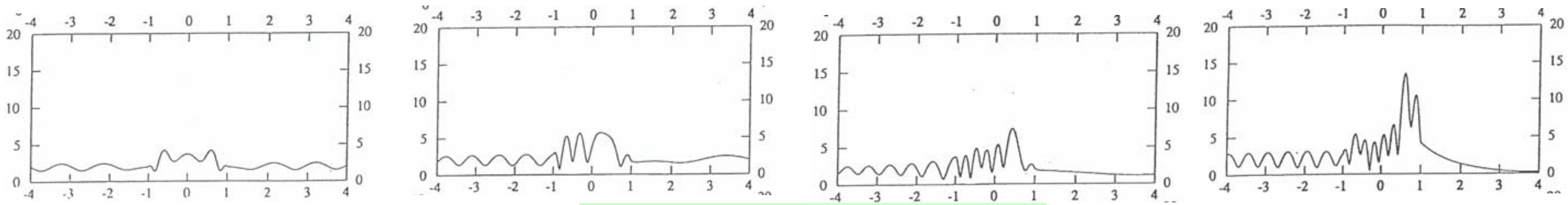


$$Amplitude = \sqrt{\mathrm{Re}^2(w) + \mathrm{Im}^2(w)}$$

$\gamma = 0^\circ$ $\gamma = 30^\circ$ $\gamma = 60^\circ$ $\gamma = 90^\circ$



Present method $\eta = 2, \mu^I / \mu^M = 1/6, \rho^I / \rho^M = 2/3$

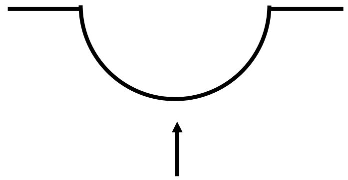


vertical

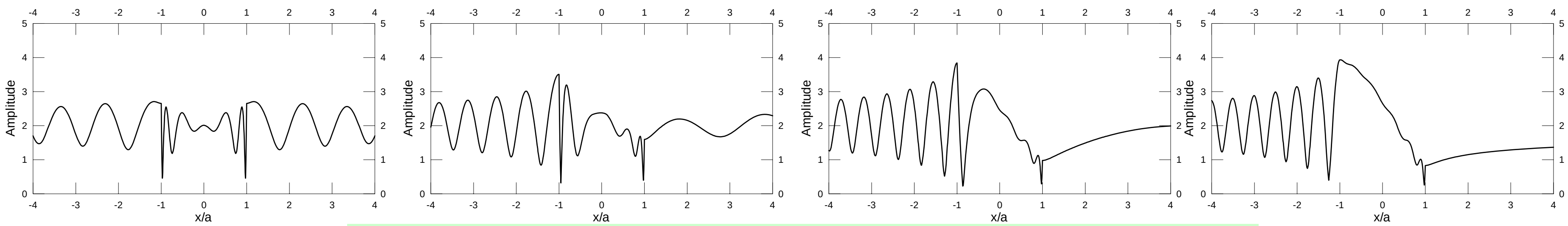
Manoogian's results [60]

horizontal

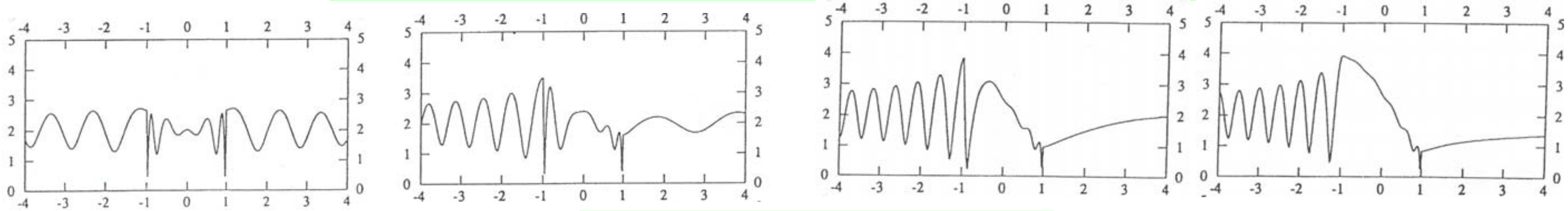
Limiting case of a canyon



$\gamma = 0^{\circ}$ $\gamma = 30^{\circ}$ $\gamma = 60^{\circ}$ $\gamma = 90^{\circ}$



Present method $\eta = 2, \mu^{I} / \mu^{M} = 10^{-8}, \rho^{I} / \rho^{M} = 2/3$

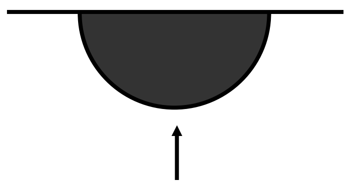


Manoogian's results [60]

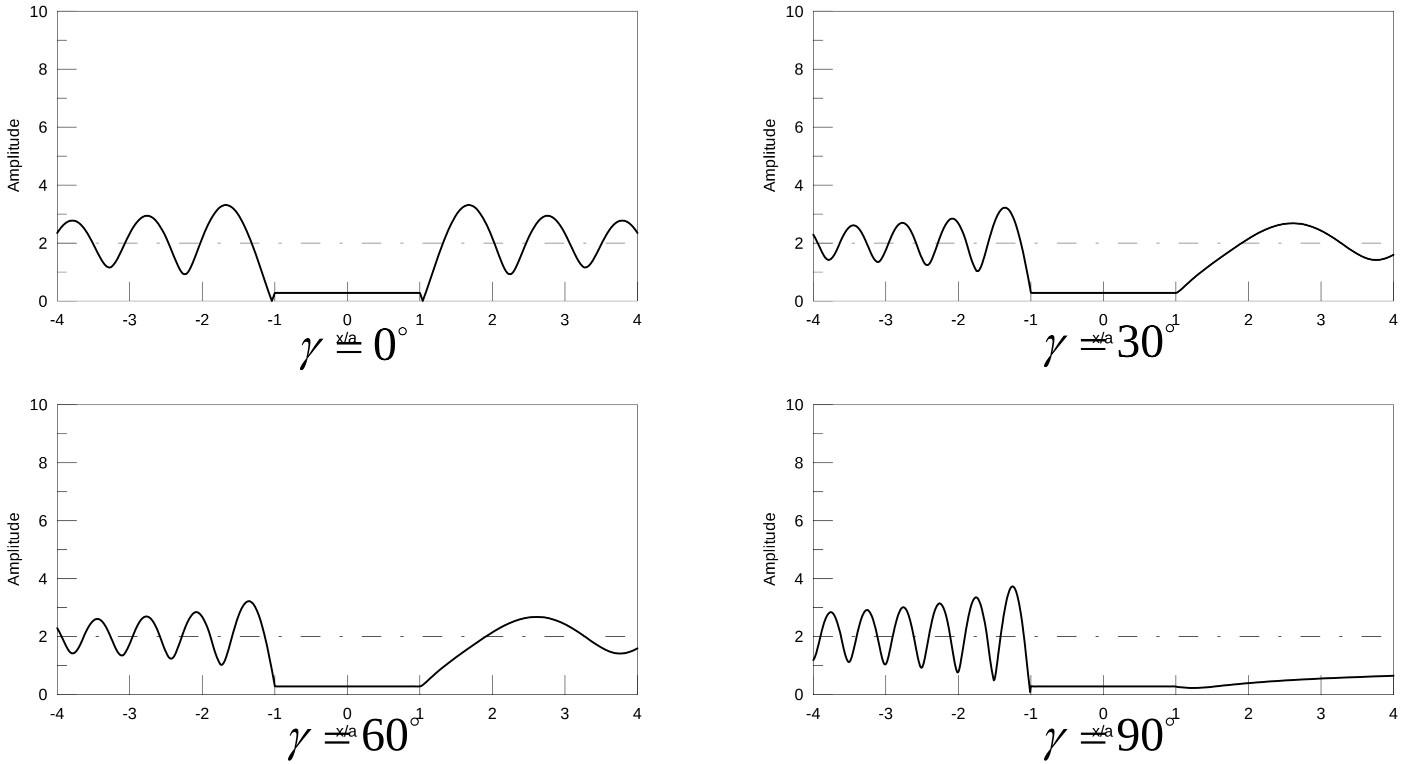
vertical

horizontal

Limiting case of a rigid alluvial valley

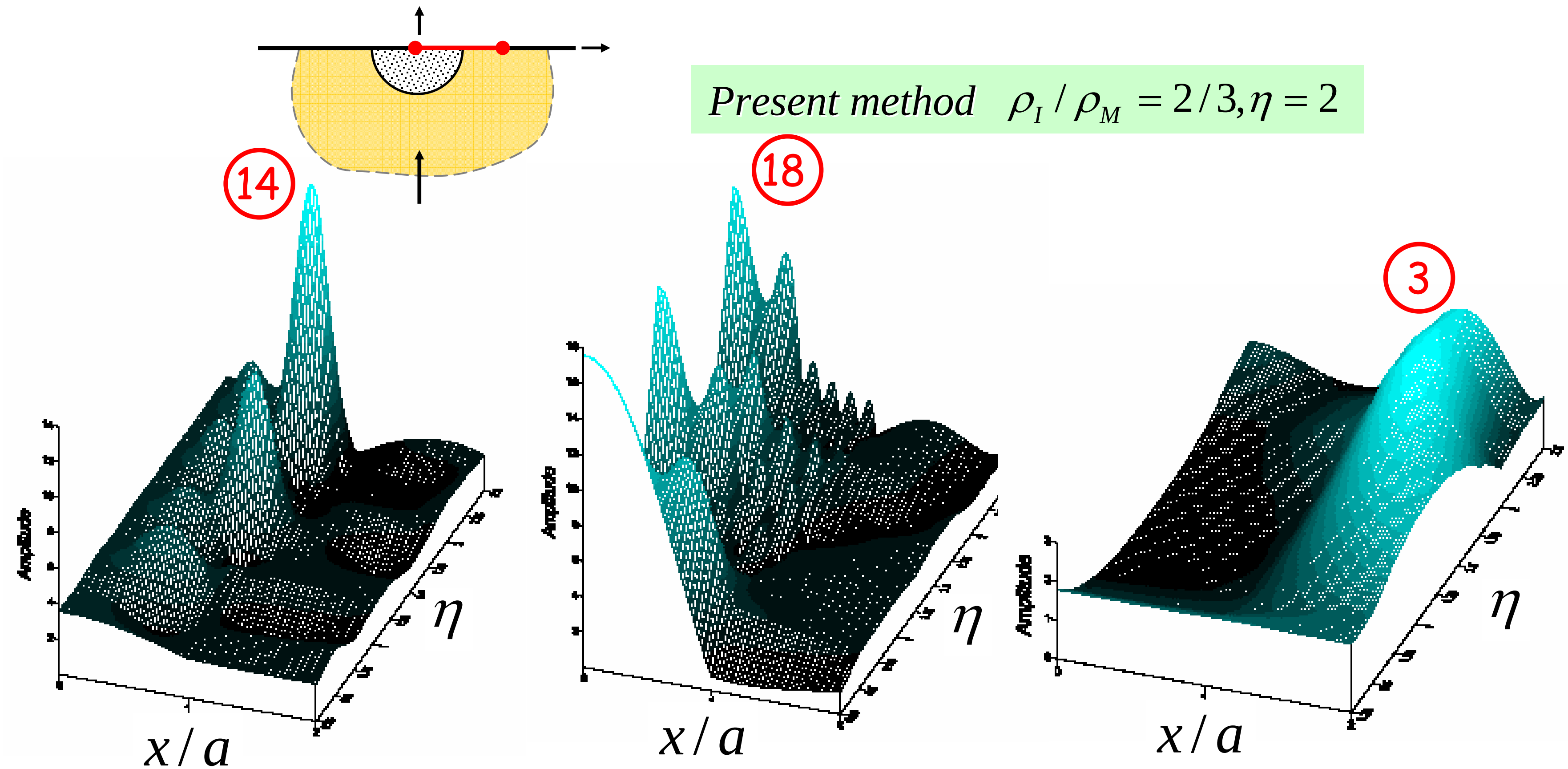


Present method $\eta = 2, \mu^I / \mu^M = 10^4, \rho^I / \rho^M = 2/3$



$\gamma = 0^\circ$ $\gamma = 30^\circ$ $\gamma = 60^\circ$ $\gamma = 90^\circ$

Soft-basin effect

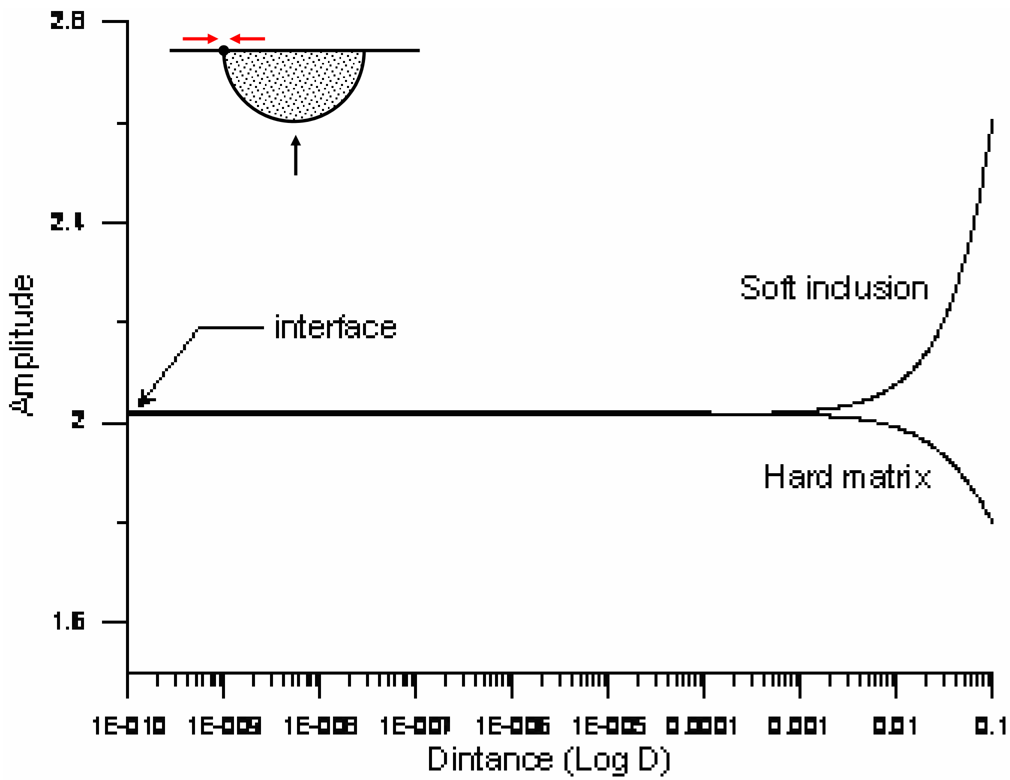


$c_I / c_M = 3/6$

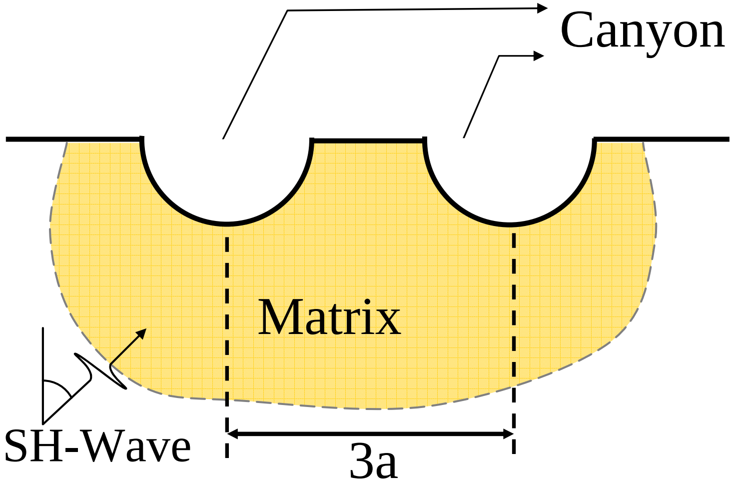
$c_I / c_M = 2/6$

$c_I / c_M = 12/6$

No boundary-layer effect



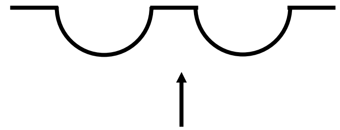
A half-plane problem with two alluvial valleys subject to the incident SH-wave



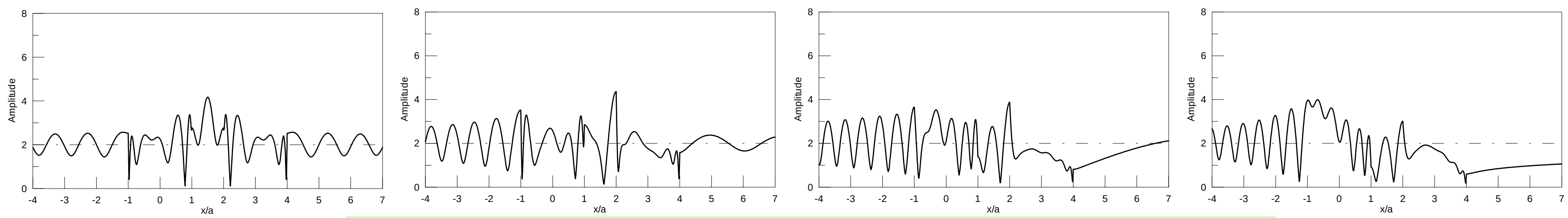
房[93]將正弦和餘弦函數的正交特性使用錯誤，以至於推導出錯誤的聯立方程，求得錯誤的結果。

--亞太學報　曹2004

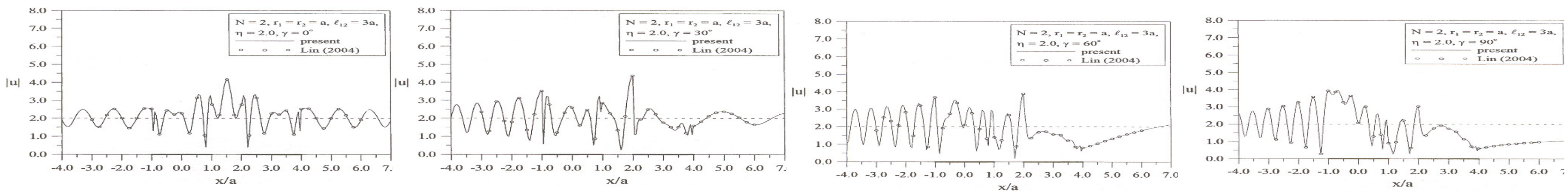
Limiting case of two canyons



$\gamma = 0^\circ$ $\gamma = 30^\circ$ $\gamma = 60^\circ$ $\gamma = 90^\circ$



Present method $\eta = 2, \mu^I / \mu^M = 10^{-8}, \rho^I / \rho^M = 2/3$

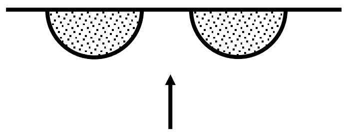


Tsaur et al.'s results [103]

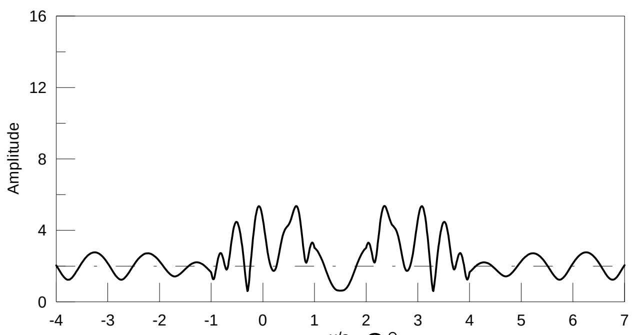
vertical

horizontal

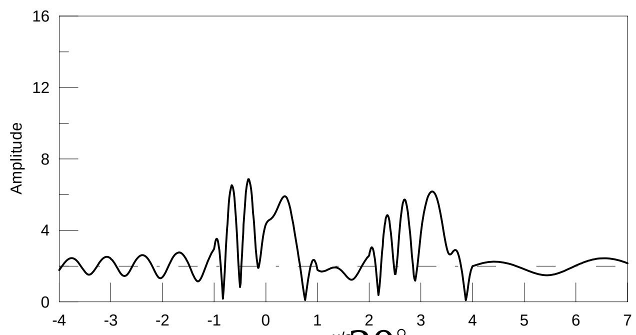
Surface displacements of two alluvial valleys



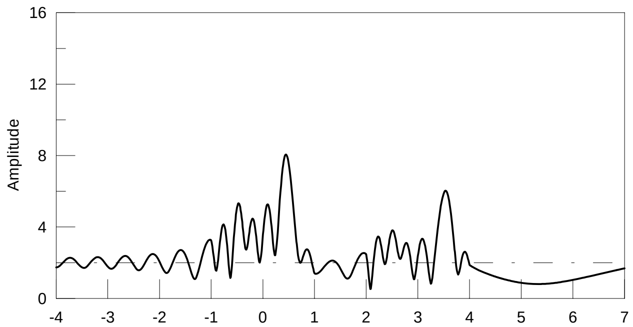
Present method $\eta = 2, \mu^I/\mu^M = 1/6, \rho^I/\rho^M = 2/3$



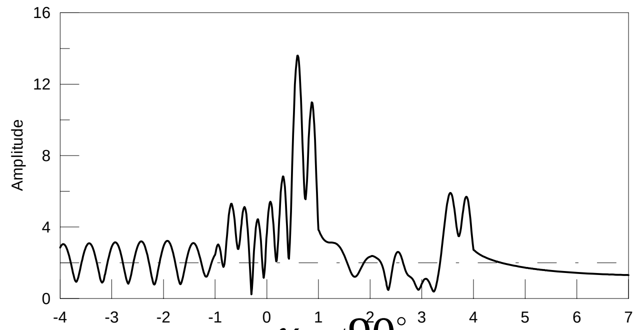
$\gamma = 0^\circ$



$\gamma = 30^\circ$

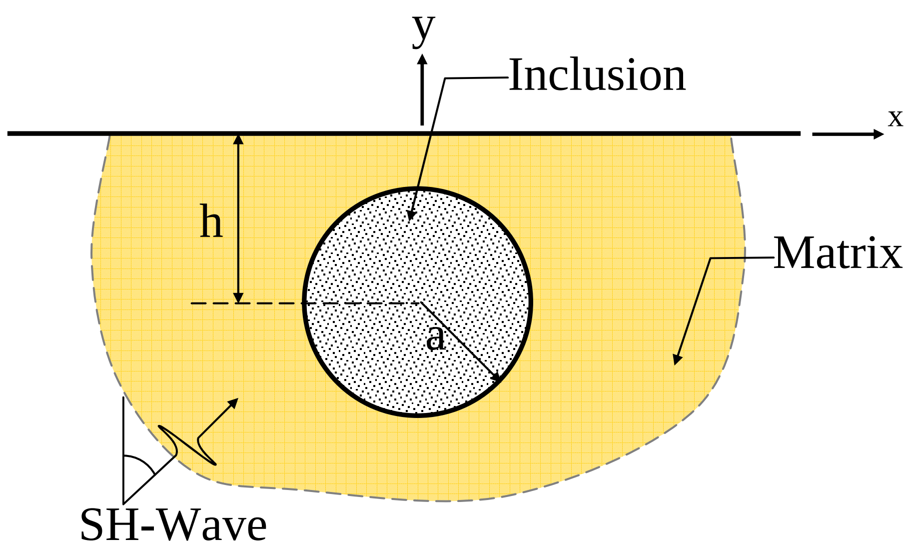


$\gamma = 60^\circ$

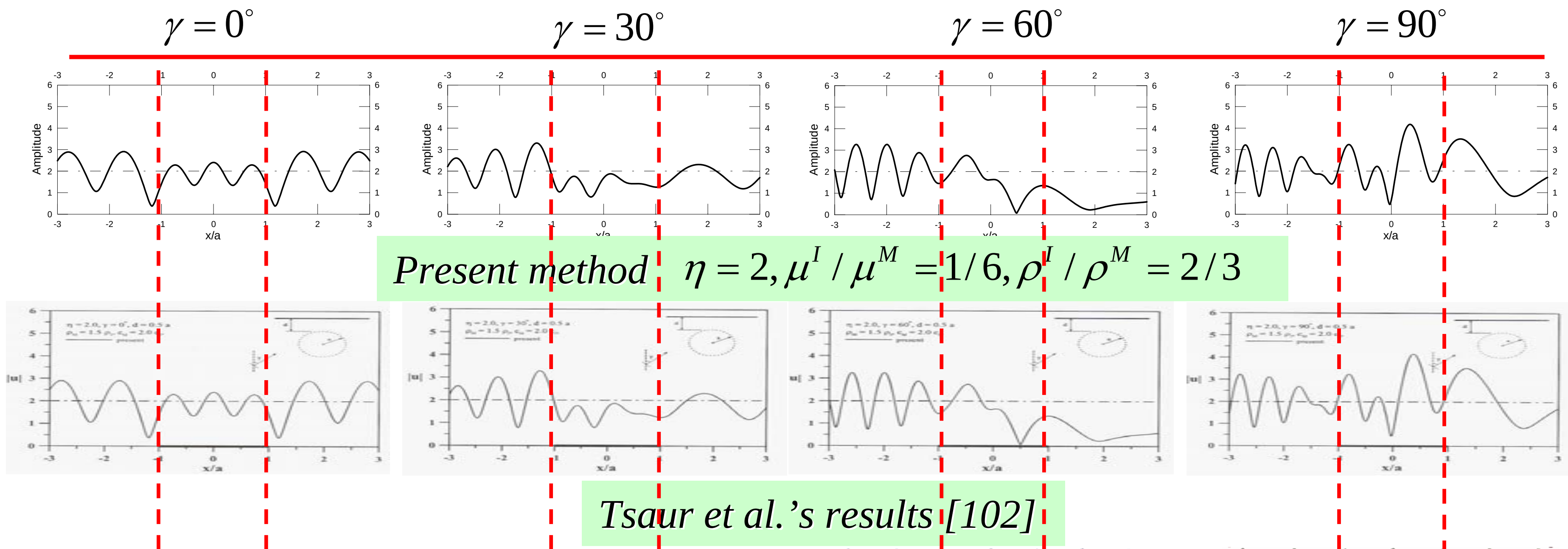


$\gamma = 90^\circ$

A half-plane problem with a circular inclusion subject to the incident SH-wave



Surface displacements of a inclusion problem under the ground surface

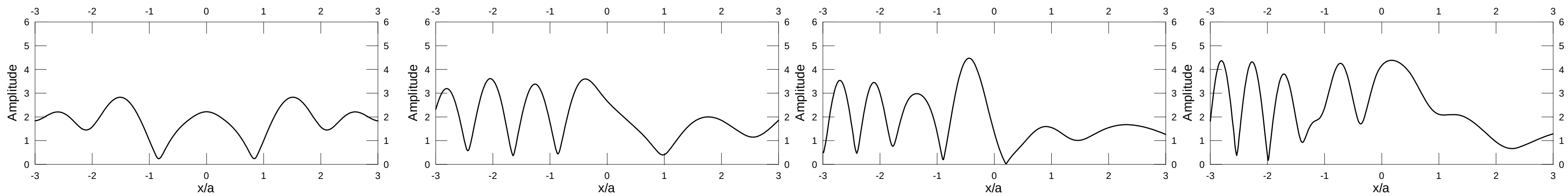


When I solved this problem I could find no published results for comparison. I also verified my results using the limiting cases. I did not have the benefit of published results for comparing the intermediate cases. I would note that due to precision limits in the Fortran compiler that I was using at the time.

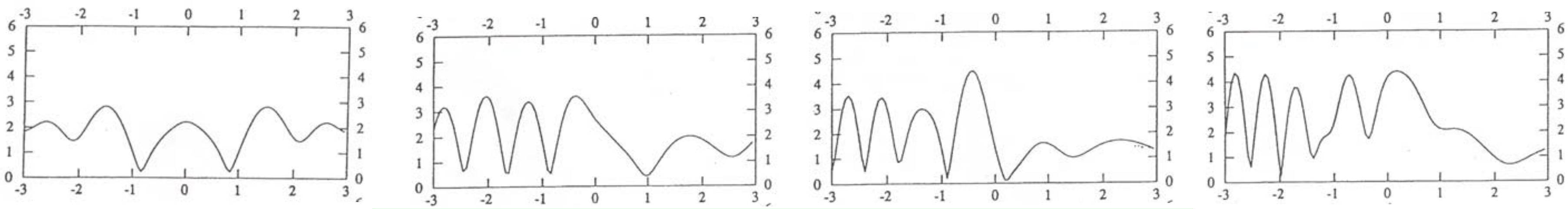
--Private communication

Limiting case of a cavity problem

$\gamma = 0^{\circ}$ $\gamma = 30^{\circ}$ $\gamma = 60^{\circ}$ $\gamma = 90^{\circ}$



Present method $\eta = 2, \mu^{I} / \mu^{M} = 10^{-8}, \rho^{I} / \rho^{M} = 2/3$

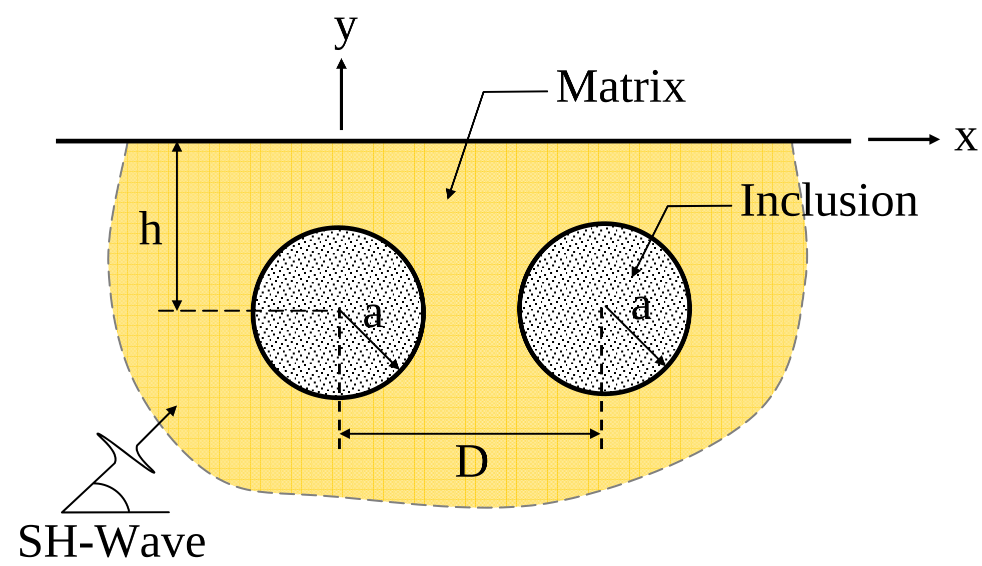


Lee and Manoogian's [53] for the cavity case.

vertical

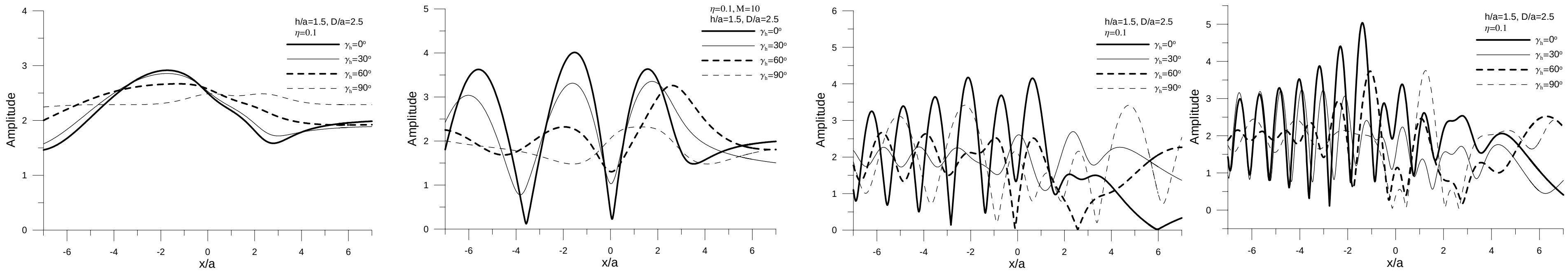
horizontal

A half-plane problem with two circular inclusions subject to the SH-wave

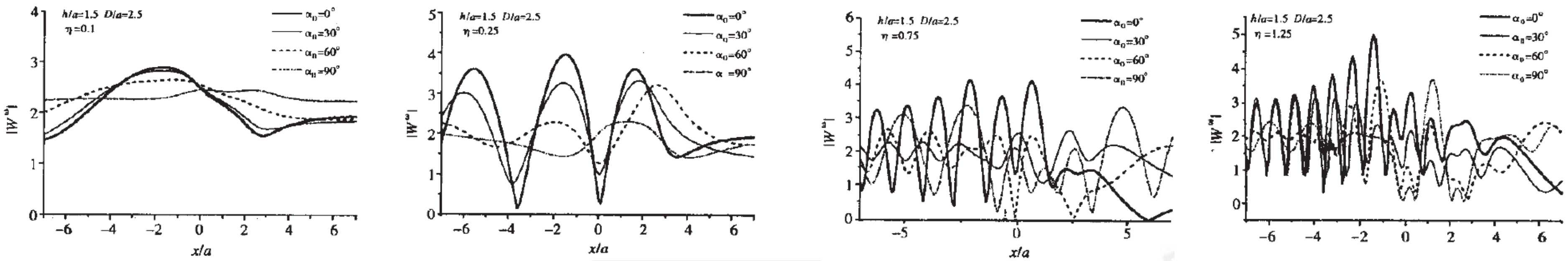


Limiting case of two-cavities problem

$\eta = 0.1$ $\eta = 0.25$ $\eta = 0.75$ $\eta = 1.25$



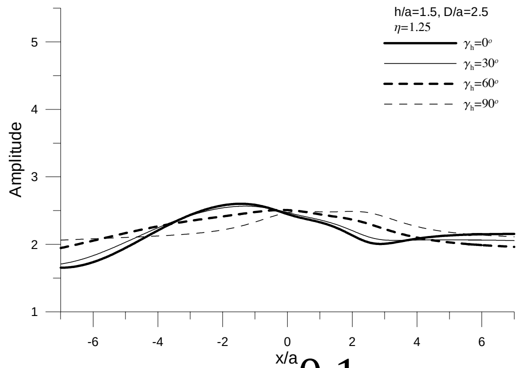
Present method $\mu^I / \mu^M = 10^{-8}, \rho^I / \rho^M = 2/3, D/a = 2.5$



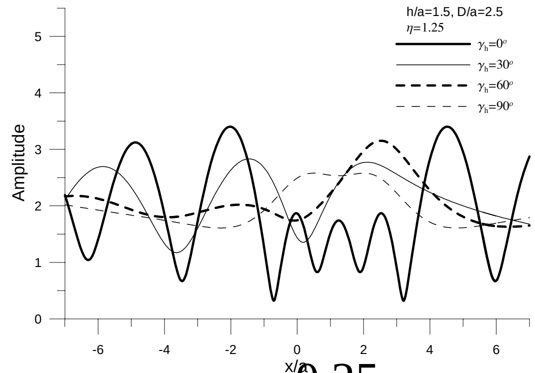
Jiang et al. result [95]

Surface amplitudes of two-inclusions problem

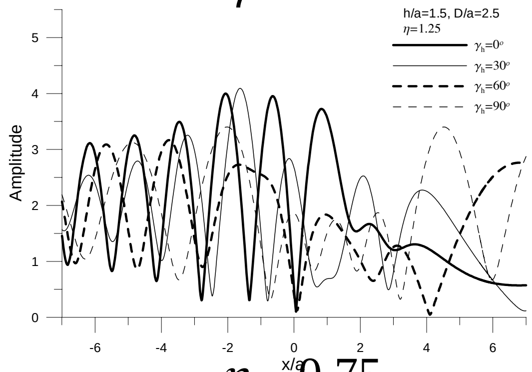
Present method $\mu^I / \mu^M = 1/6, \rho^I / \rho^M = 2/3, D/a = 2.5$



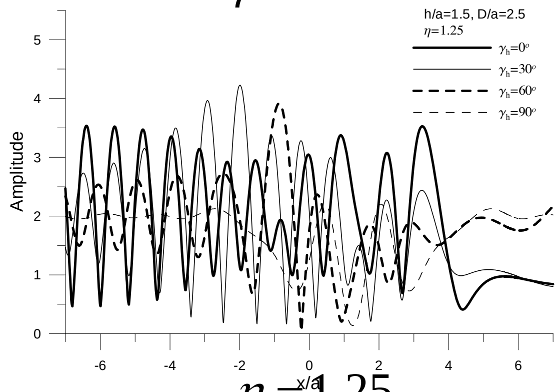
$\eta = 0.1$



$\eta = 0.25$



$\eta = 0.75$

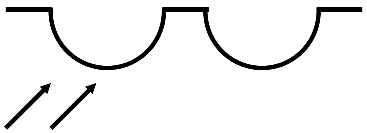


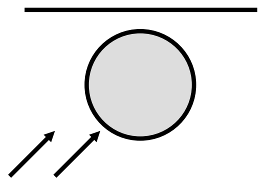
$\eta = 1.25$

Conclusions

- A systematic way to solve the Helmholtz problems with circular boundaries was proposed successfully in this paper by using the null-field integral equation in conjunction with degenerate kernels and Fourier series.
- The present method is more general for calculating the torsion and bending problems with arbitrary number of holes and various radii and positions than other approach.
- Our approach can deal with the cavity problem as a limiting of inclusion problem with zero shear modulus.

Some findings





房營光[1995] *Analytical solution*	*Lee & Manoogian*[1992]
Tsaur et al.[2004] *Analytical solution*	*Present method*
Present method	*Tsaur et al.*

?

Thanks for your kind attentions.

You can get more information from our website.

http://msvlab.hre.ntou.edu.tw/