Elimination of spurious eigenfrequency in the boundary element method using CHEEF technique

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Abstract

It was found that the imaginary-part BEM for eigenproblems results in spurious eigensolutions. By adding the constraints from the null-field integral equation, the CHEEF method (Combined Helmholtz Exterior integral Equation Formulation) is proposed to eliminate spurious eigenfrequencies. The circular cavity is demonstrated to check the validity of the proposed method.
Keywords: Imaginary-part BEM; CHEEF; SVD techniques; Spurious eigenvalue

摘要

使用虛部核函數之邊界元素法來求解 Helmholtz 特徵值問題時，會有假根問題產生。為了解決假根問題，我們提出了一套 CHEEF 法來過濾假根。由於缺少實部的束制條件，可由零場外域積分方程取得額外的束制條件來補足不足的束制條件。本文以一個圓形的例子來驗證 CHEEF 法之可行性。
關鍵字: 虛部邊界元素法; CHEEF; 奇異值分解法; 假根

1. Introduction

Based on the integral equations for the eigenproblem, the BEMs have been utilized to solve the interior and exterior problems for a long time. By

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employing the complex-valued BEM [1], the eigenvalues and eigenmodes for the eigenproblem can be determined. However, complex-valued computation is time consuming and not simple. Nowak and Neves [2] proposed a multiple reciprocity method (MRM) in the real-valued computation only. To avoid complex-valued computation, the simplified method by using only the real-part or imaginary-part kernel was presented by De Mey [3]. Later, Hutchinson also utilized the real-part kernel to solve the membrane [4] and plate eigenproblems [5]. Although the complex-valued computation was avoided, they must face the occurrence of spurious eigenvalues. Shaw [6] commented that only the real-part formulation was incorrect since the eigenequation must satisfy the real-part and imaginary-part equations at the same time. Niwa *et al.* [7] also stated that "One must take care to use the complete Green's function for outgoing waves, as attempts to use just the real (singular) or imaginary (regular) part separately will not provide the complete spectrum". Hutchinson [8] replied that the claim of incorrectness was perhaps a little strong since the real-part BEM does not miss any true eigenvalue although the solution is contaminated by spurious ones according to his numerical experience. However, no proof was provided. Chen and his coworkers [9] have derived the true and spurious eigenvalues for circular problems by using the degenerate kernels and circulants. Hutchinson [8] proposed detection technique by examining the modal shapes. Nevertheless, this technique may fail in some cases which have been discussed by Chen *et al.* [10]. A systematical technique to sort out spurious solution is not trivial. The complex-valued BEM may waste too much unnecessary calculation. However, either real-part or imaginary-part BEM results in spurious eigenvalues [10, 11]. A more efficient method using CHEEF concept will be addressed here for imaginary-part BEM according to the successful experience of real-part BEM [11].

In this paper, we will employ the CHEEF method to filter out the spurious eigenvalues for eigenproblems in the boundary element method. The position where to place the CHEEF point efficiently will be studied analytically and verified numerically. The optimum number of extra equations for the CHEEF points will also be discussed at the same time. After combining the influence matrix with the CHEEF equations, the SVD technique will be utilized to determine the eigenvalues, multiplicity and boundary modes. The boundary modes can be easily extracted from the right unitary matrix in SVD. The circular cavity will be demonstrated analytically and numerically to check the validity of the proposed method.

2. Imaginary-part BEM in conjunction with CHEEF technique for 2-D acoustic eigenproblem

The governing equation for an eigenproblem is the Helmholtz equation as follows:

$$(\nabla^2 + k^2)u(x) = 0, \quad x \in D. \tag{1}$$

where ∇^2 is the Laplacian operator, D is the domain of the cavity and k is the wave number which is the angular frequency over the speed of sound. The boundary conditions can be either the Dirichlet or Neumann type. Based on the dual formulation (singular and hypersingular formulation), the boundary integral equation for smooth boundary points are represented as

$$\pi u(x) = C.P.V.\int_B T(s,x)u(s)dB(s) - R.P.V.\int_B U(s,x)t(s)dB(s), \quad x \in B \tag{2}$$

$$\pi t(x) = H.P.V.\int_B M(s,x)u(s)dB(s) - C.P.V.\int_B L(s,x)t(s)dB(s), \quad x \in B \tag{3}$$

where $C.P.V.$, $R.P.V.$ and $H.P.V.$ denote the Cauchy principal value, the Riemann principal value and Hadamard principal value, respectively; x is the field point and s is the source point, $t(s) = \frac{\partial u(s)}{\partial n_s}$, $U(s,x)$ is the fundamental solution, $T(s,x) = \frac{\partial U(s,x)}{\partial n_s}$, $L(s,x) = \frac{\partial U(s,x)}{\partial n_x}$ and $M(s,x) = \frac{\partial^2 U(s,x)}{\partial n_s \partial n_x}$, B denotes the boundary enclosing D. Here, we choose only the imaginary-part fundamental solution for the kernel function. The closed form of imaginary-part fundamental solution is shown below:

$$U_I(s,x) = \mathrm{Im}\{iH_0^{(1)}(kr)\} = J_0(kr), \tag{4}$$

where $H_0^{(1)}(kr)$ and $J_0(kr)$ denote the zeroth order of the first kind Hankel function and Bessel function, respectively, and *Im* denotes the imaginary part. Eqs. (2) and (3) are reduced to

$$0 = \int_B T_I(s,x)u(s)dB(s) - \int_B U_I(s,x)t(s)dB(s), \quad x \in B, \tag{5}$$

$$0 = \int_B M_I(s,x)u(s)dB(s) - \int_B L_I(s,x)t(s)dB(s), \quad x \in B, \tag{6}$$

By discretizing the boundary into $2N$ constant elements, Eqs (5) and (6) are written as

$$\{0\} = [T_I]_{2N\times 2N}\{u\}_{2N\times 1} - [U_I]_{2N\times 2N}\{t\}_{2N\times 1}, \tag{7}$$

$$\{0\} = [M_I]_{2N\times 2N}\{u\}_{2N\times 1} - [L_I]_{2N\times 2N}\{t\}_{2N\times 1}, \tag{8}$$

where the $[U_I]$, $[T_I]$, $[L_I]$ and $[M_I]$ matrices are the corresponding influence coefficient matrices resulting from the U, T, L and M kernels, respectively. Null-field integral equation of CHEEF point yields

$$[T_I^C]_{a\times 2N}\{u\}_{2N\times 1} = \{0\}. \tag{9}$$

For the Neumann problem, Eqs. (7) and (8) merge to

$$[C]_{(2N+a)\times 2N}\{u\}_{2N\times 1} = \{0\}, \tag{10}$$

where

$$[C]_{(2N+a)\times 2N} = \begin{bmatrix} [T_I]_{2N\times 2N} \\ [T_I^C]_{a\times 2N} \end{bmatrix}, \tag{11}$$

By employing the SVD technique for [C], we can plot the minimum singular value versus the wave number (k) and find the eigenvalue from the drop location. The analytical results of eigenequation, boundary eigen mode and null-field integral equation are summarized in Tables 1 and 2 for imaginary-part UT and LM BEMs, respectively.

3. Numerical example

We considered a circular cavity with radius 1 *m* subjected to Neumann boundary condition to check the validity of the CHEEF method. Twelve elements were adopted in the boundary element mesh. Fig. 1 shows the minimum singular value (σ_1) versus the wave number (k) where the true and spurious eigenvalues are obtained using the imaginary-part UT BEM. Fig. 2 shows the the minimum singular value (σ_1) versus the wave number (k) where only the true eigenvalues are obtained using the imaginary-part UT BEM in conjunction with CHEEF method.

4. Conclusions

The CHEEF method in conjunction with the SVD technique was proposed to determine the true eignevalues. If the CHEEF points were properly chosen, the spurious eigenvalues can be sorted out. A circular case was demonstrated to see the validity of the present formulation.

5. References

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Table 1 True and spurious eigenequations using the imaginary-part UT BEM

	Eigenequation	Boundary Eigenmode	Interior Mode $u(\rho,\phi)$ $(0 \le \rho \le a)$ $(0 \le \phi < 2\pi)$	Null-field integral equation
Dirichlet Problem	True $J_n(ka)=0$	$C_n \cos(n\phi)$	$J_n(ka)\, J_n(k\rho) \cos(n\phi)$	$C_n(J_n(k\rho)\, J_n(ka))\cos(n\phi)=0$ (UT) $C_n(J'_n(k\rho)\, J_n(ka))\cos(n\phi)=0$ (LM)
	Spurious $J_n(ka)=0$	$C_n \cos(n\phi)$	$J_n(ka)\, J_n(k\rho) \cos(n\phi)$	$C_n(J_n(k\rho)\, J_n(ka))\cos(n\phi)=0$ (UT) $C_n(J'_n(k\rho)\, J_n(ka))\cos(n\phi)=0$ (LM)
Neumann Problem	True $J'_n(ka)=0$	$C_n \cos(n\phi)$	$J'_n(ka)\, J_n(k\rho) \cos(n\phi)$	$C_n(J_n(k\rho)\, J'_n(ka))\cos(n\phi)=0$ (UT) $C_n(J'_n(k\rho)\, J'_n(ka))\cos(n\phi)=0$ (LM)
	Spurious $J_n(ka)=0$	$C_n \cos(n\phi)$	$J'_n(ka)\, J_n(k\rho) \cos(n\phi)$	$C_n(J_n(k\rho)\, J'_n(ka))\cos(n\phi)=0$ (UT) $C_n(J'_n(k\rho)\, J'_n(ka))\cos(n\phi)=0$ (LM)

Table 2 True and spurious eigenequations using the imaginary-part LM BEM

	Eigenequation	Boundary Eigenmode	Interior Mode $u(\rho,\phi)$ $(0 \le \rho \le a)$ $(0 \le \phi < 2\pi)$	Null-field integral equation
Dirichlet Problem	True $J_n(ka)=0$	$C_n \cos(n\phi)$	$J_n(ka)\, J'_n(k\rho) \cos(n\phi)$	$C_n(J_n(k\rho)\, J_n(ka))\cos(n\phi)=0$ (UT) $C_n(J'_n(k\rho)\, J_n(ka))\cos(n\phi)=0$ (LM)
	Spurious $J'_n(ka)=0$	$C_n \cos(n\phi)$	$J_n(ka)\, J'_n(k\rho) \cos(n\phi)$	$C_n(J_n(k\rho)\, J_n(ka))\cos(n\phi)=0$ (UT) $C_n(J'_n(k\rho)\, J_n(ka))\cos(n\phi)=0$ (LM)
Neumann Problem	True $J'_n(ka)=0$	$C_n \cos(n\phi)$	$J'_n(ka)\, J'_n(k\rho) \cos(n\phi)$	$C_n(J_n(k\rho)\, J'_n(ka))\cos(n\phi)=0$ (UT) $C_n(J'_n(k\rho)\, J'_n(ka))\cos(n\phi)=0$ (LM)
	Spurious $J'_n(ka)=0$	$C_n \cos(n\phi)$	$J'_n(ka)\, J'_n(k\rho) \cos(n\phi)$	$C_n(J_n(k\rho)\, J'_n(ka))\cos(n\phi)=0$ (UT) $C_n(J'_n(k\rho)\, J'_n(ka))\cos(n\phi)=0$ (LM)

where $n = 0, 1, 2, \cdots\cdots$

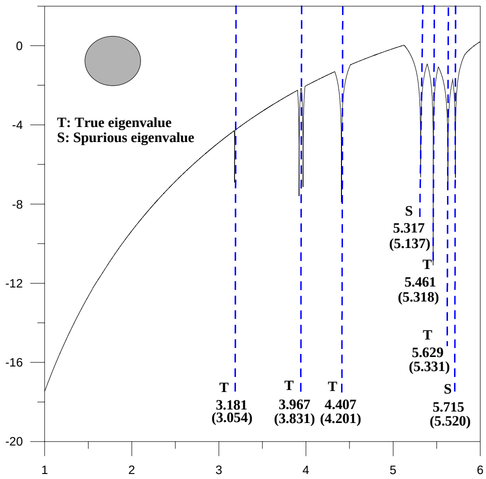

Fig. 1 The minimum singular value σ_1 versus the wave number *k* by using UT formulation.

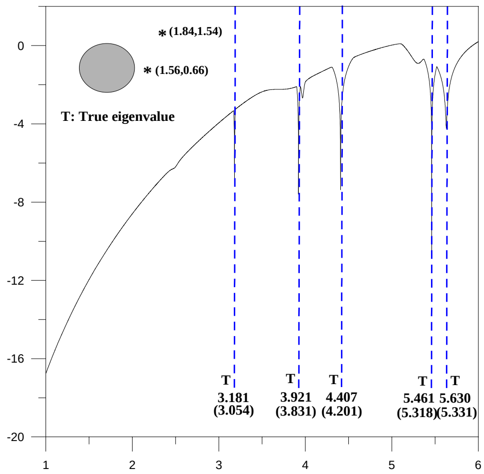

Fig. 2 The minimum singular value σ_1 versus the wave number *k* by using UT formulation with two CHEEF points.